EOB MODELS FOR COALESCING BINARIES

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The IHES effective-one-body (EOB) code: eob.ihes.fr
T. Damour, AN,
S. Bernuzzi
D. Bini, P. Fleig

A. Nagar, 24 May 2016 - Hannover

Theory: SYNERGY between

Analytical and Numerical General Relativity

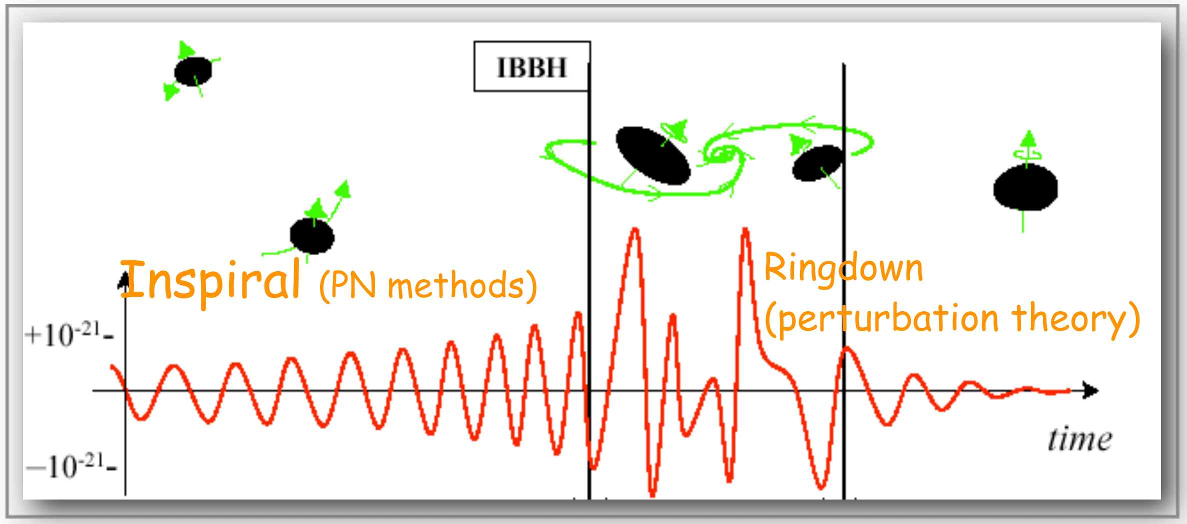
(AR/NR)

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

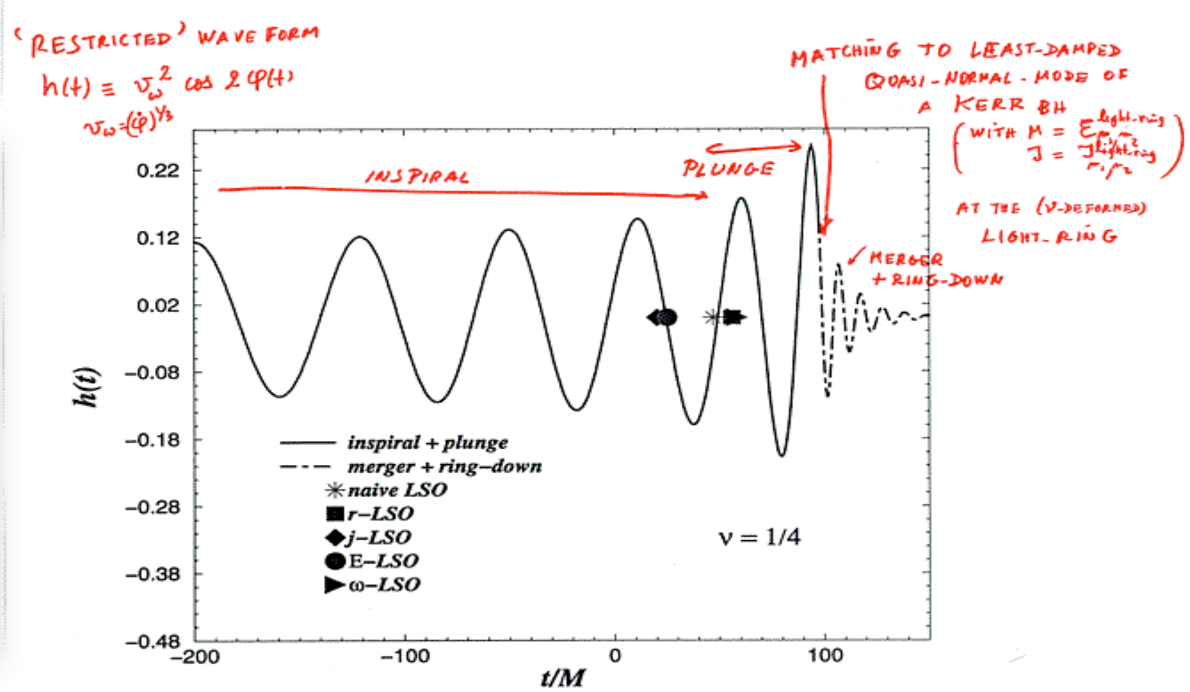
EOBNR

TEMPLATES FOR GWS FROM BBH COALESCENCE

Brady, Craighton & Thorne, 1998



Merger: Numerical Relativity (?)

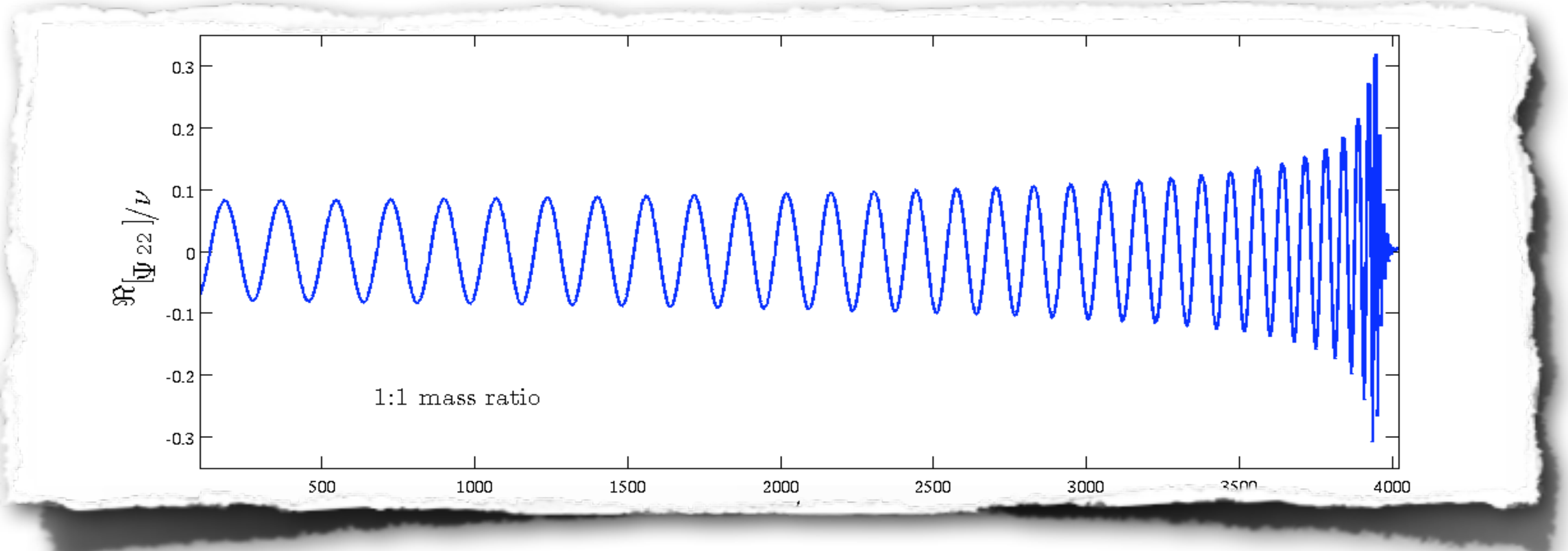


Effective-One-Body (Buonanno & Damour (2000))

PN-resummation (Damour, Iyer & Sathyaprakash (1998))

Numerical Relativity: >= 2005 (F. Pretorius, Campanelli et al., Baker et al.)

Most accurate data: Caltech-Cornell spectral code: M. Scheel et al., 2008 (SXS collaboration)



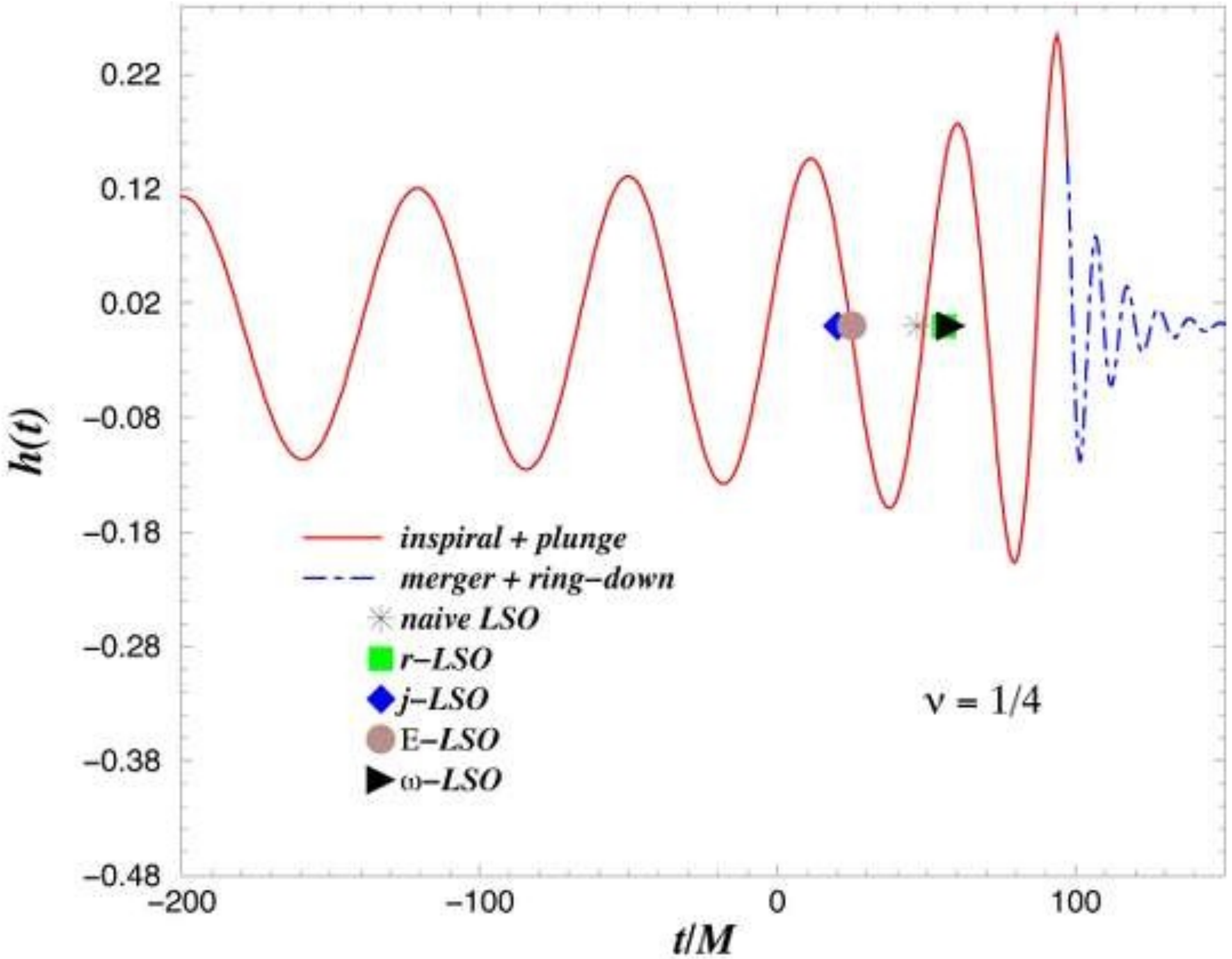
Phase error:

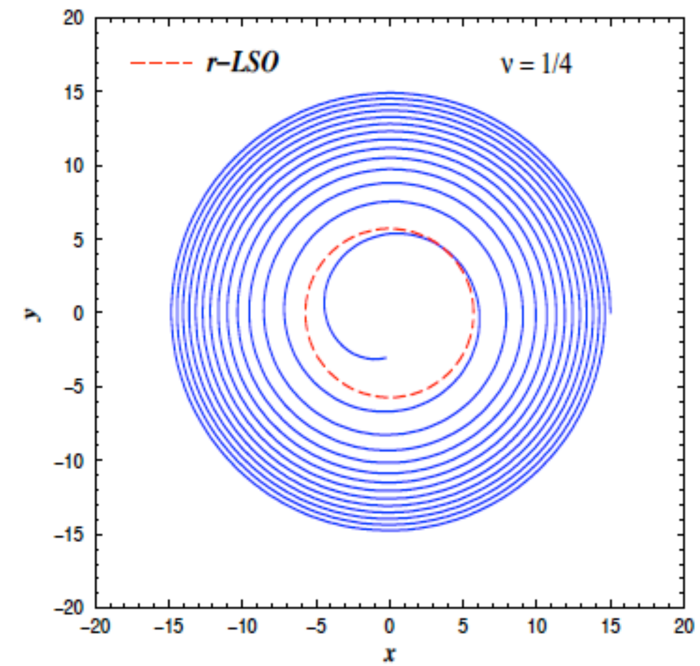
< 0.02 rad (inspiral)

<0.1 rad (ringdown)

EFFECTIVE ONE BODY (EOB): 2000

5 years before Numerical Relativity (NR):
EOB formalism was predictive, qualitatively and semi-quantitatively correct (10%)





A. Buonanno & T. Damour, PRD 59 (1999) 084006
A. Buonanno & T. Damour, PRD 62 (2000) 064015

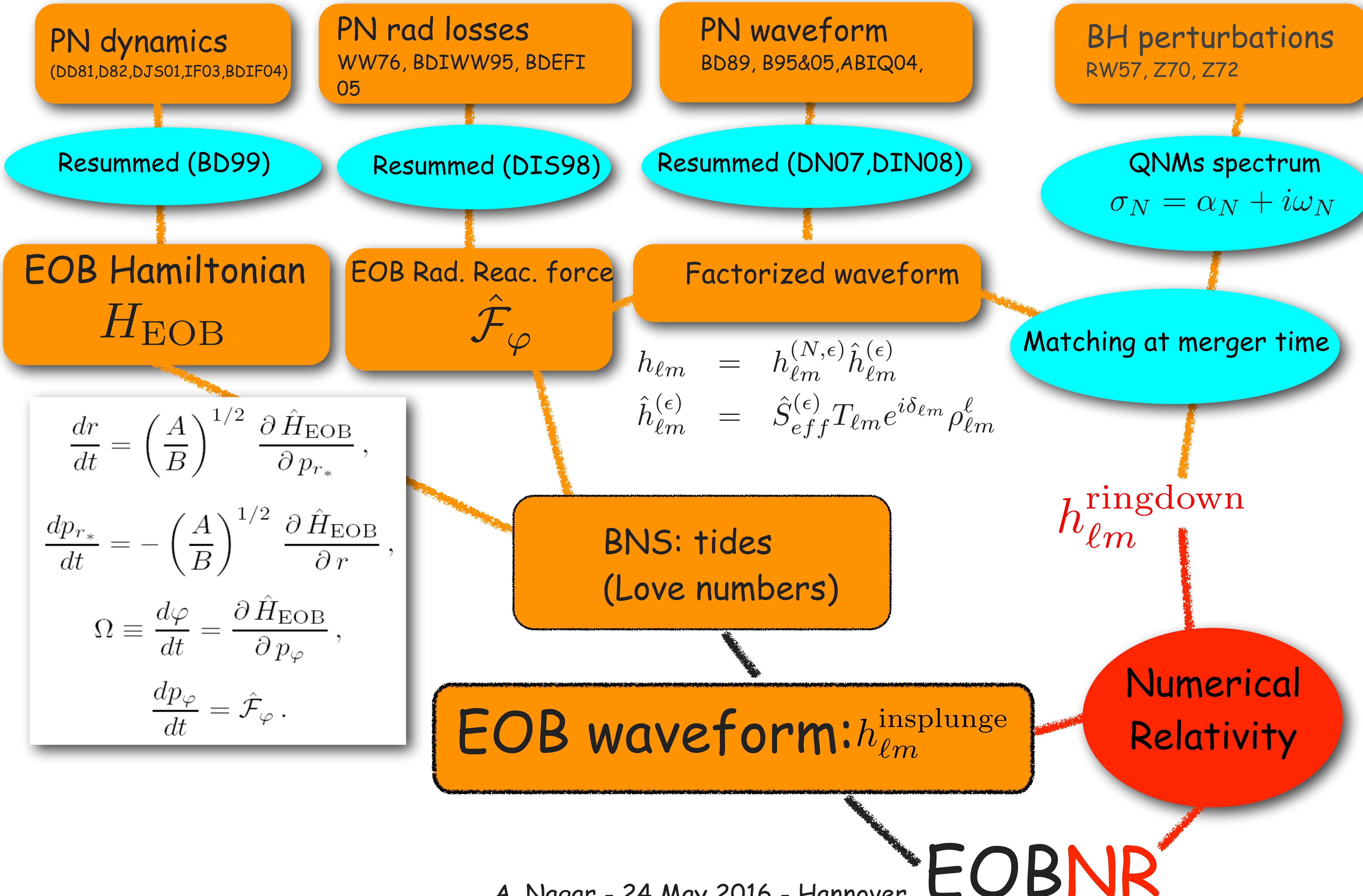
- 2-body problem into effective problem
- relative dynamics in CoM frame
- Deformation of test-particle on Schwarzschild
- Resummation of PN information
- Blurred transition from inspiral to plunge
- Final black-hole mass
- Final black hole spin
- Complete waveform

> 2005: Developing EOB & interfacing with NR
2 groups did (and do) it

- A.Buonanno et al. (AEI)
- T.Damour & AN + (>2005)

$$\nu = \frac{m_1 m_2}{(m_1+m_2)^2} = \frac{\mu}{M}$$

STRUCTURE OF THE EOB FORMALISM



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HAMILTON'S EQUATIONS & RADIATION REACTION

$$\dot{r} = \left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial p_{r_*}}$$

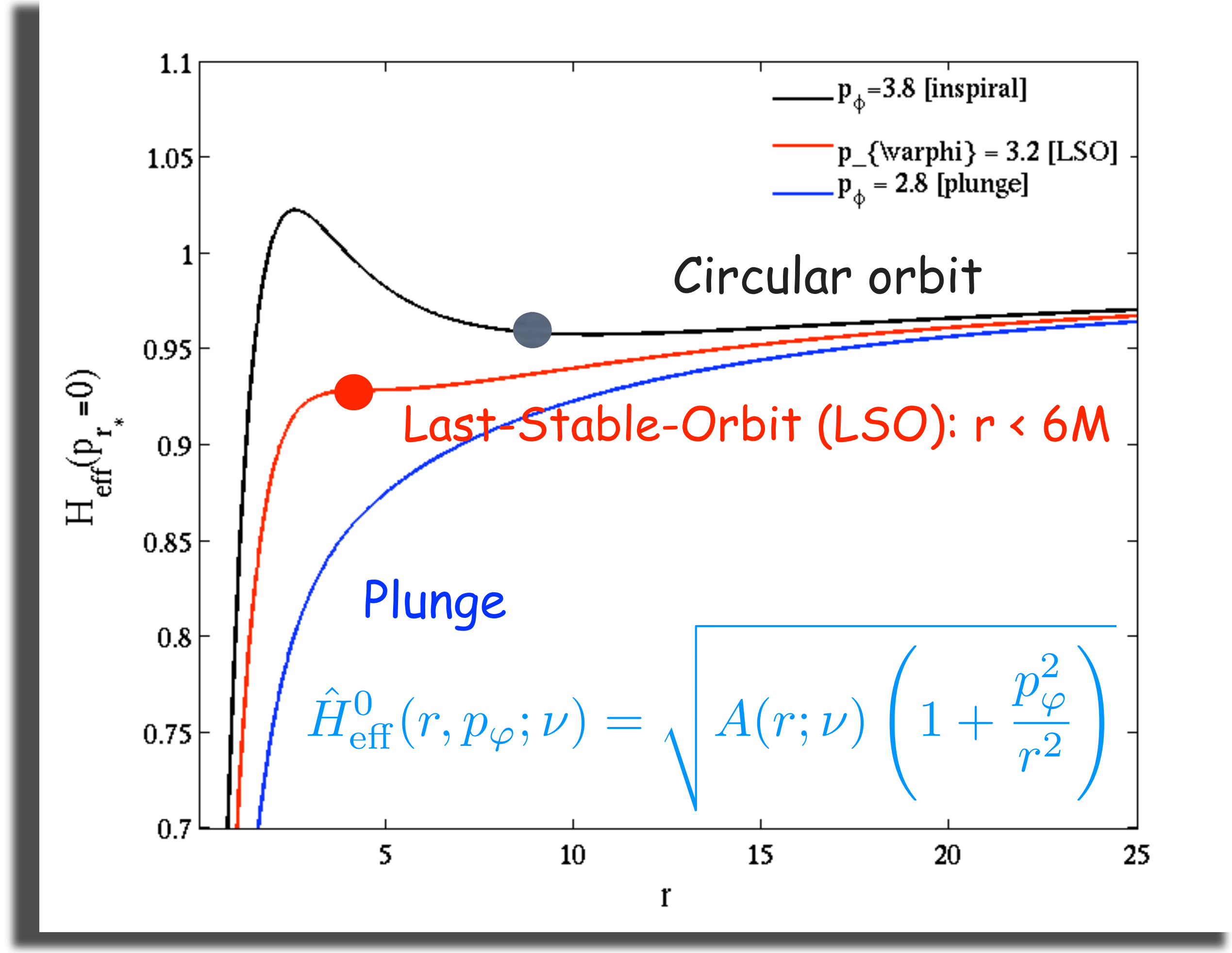
$$\dot{\varphi} = \frac{\partial \hat{H}_{\rm EOB}}{\partial p_{\varphi}} \equiv \Omega$$

$$\dot{p}_{r_*} = -\left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial r} + \hat{\mathcal{F}}_{r_*}$$

$$\dot{p}_{\varphi} = \hat{\mathcal{F}}_{\varphi}$$

$$H_{\rm EOB} = M\sqrt{1 + 2\nu\left(\hat{H}_{\rm eff} - 1\right)}$$

$$\hat{H}_{\rm eff} \equiv \sqrt{p_{r_*}^2 + A(r)\left(1 + \frac{p_{\varphi}^2}{r^2} + z_3 \frac{p_{r_*}^4}{r^2}\right)}$$



Resummation multipole by multipole

(Damour & Nagar 2007, Damour, Iyer & Nagar 2008, Damour & Nagar, 2009, Pan et al. 2011)

$$\mathcal{F}_{\varphi} \equiv -\frac{1}{8\pi\Omega} \sum_{\ell=2}^{\ell_{\rm max}} \sum_{m=1}^{\ell} (m\Omega)^2 |R h_{\ell m}^{(\epsilon)}|^2$$

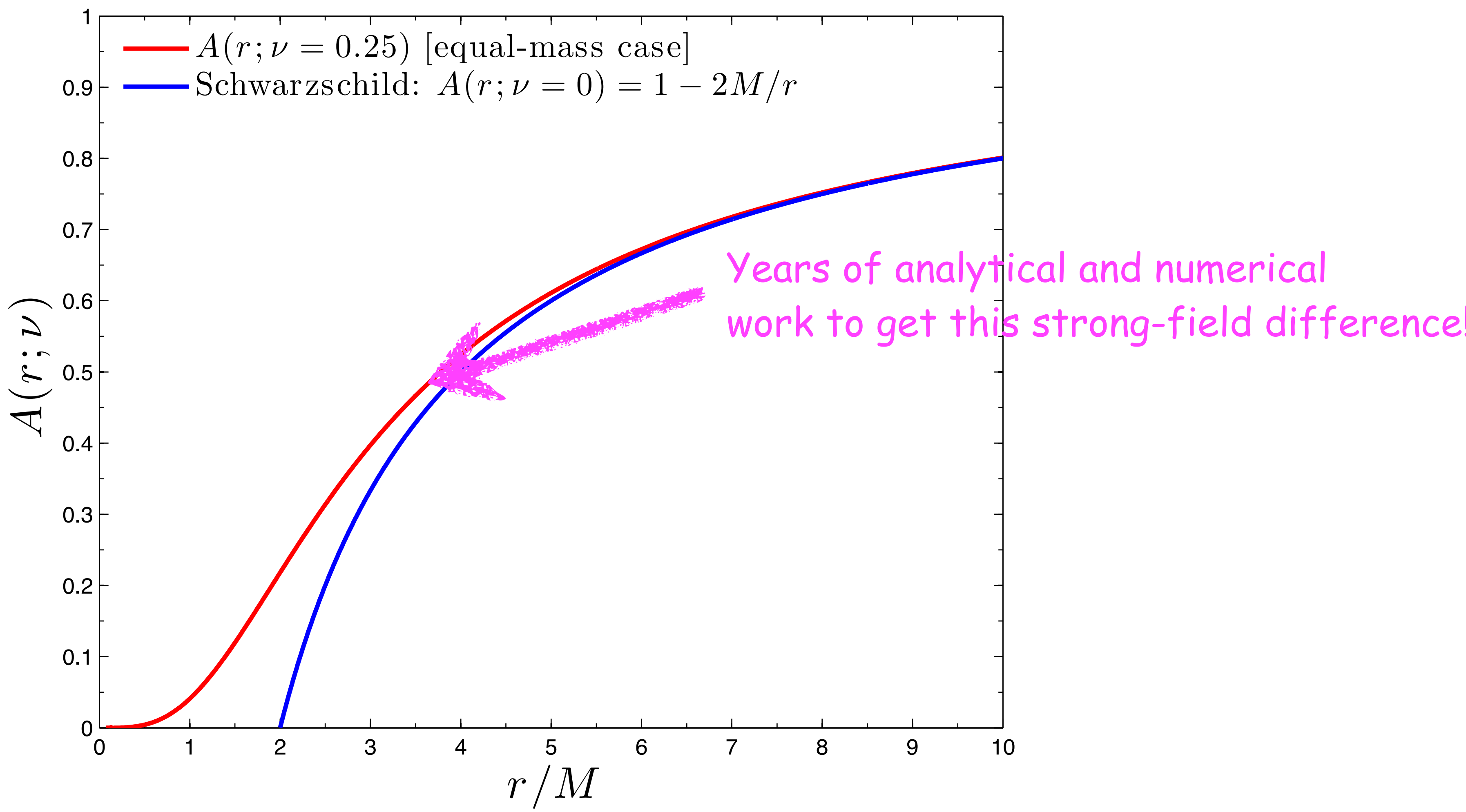
$$h_{\ell m} \equiv h_{\ell m}^{(N,\epsilon)} \hat{h}_{\ell m}^{(\epsilon)} \hat{h}_{\ell m}^{\rm NQC}$$

Newtonian × PN × NQC

THE EOB[NR] POTENTIAL

$$A_{5\mathrm{PN}}^{\mathrm{Taylor}} = 1 - 2u + 2\nu u^3 + \left(\frac{94}{3} - \frac{41}{32}\pi^2\right)\nu u^4 + \nu[a_5^c(\nu) + a_5^{\ln}\ln u]u^5 + \nu[a_6^c(\nu) + a_6^{\ln}\ln u]u^6$$

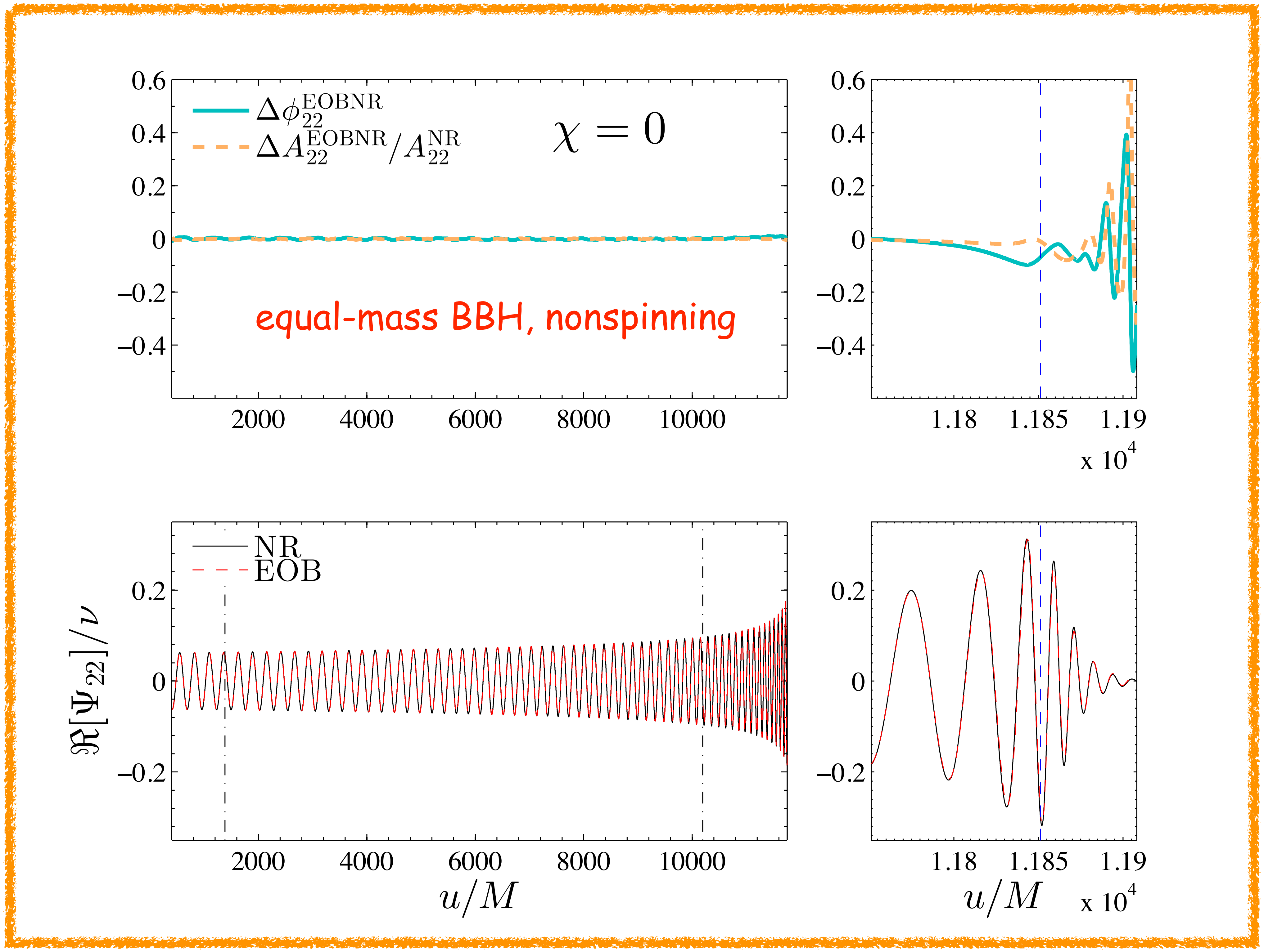
$$\nu = \frac{m_1 m_2}{(m_1+m_2)^2}$$



Padé resummation + NR calibration of $a_6^c(\nu) = 3097.3\nu^2 - 1330.6\nu + 81.3804$

$$A(u;\nu,a_6^c) = P_5^1[A_{5\mathrm{PN}}^{\mathrm{Taylor}}(u;\nu,a_6^c)]$$

RESULTS: EOBNR/NR WAVEFORMS (NO SPIN)



Analogous agreement for other (SXS) mass ratios

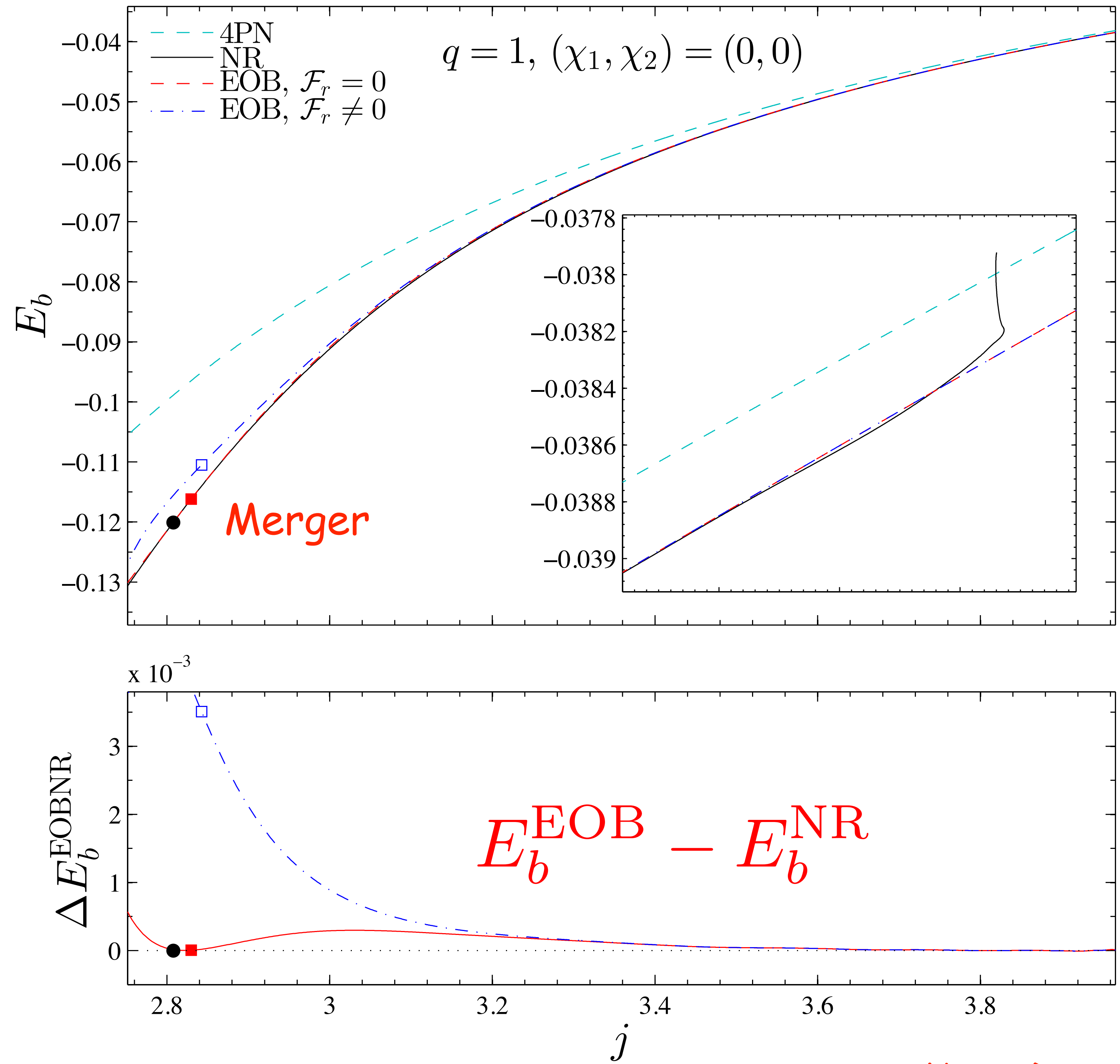
Nagar, Damour, Reisswig & Pollney, arXiv:1506.08457

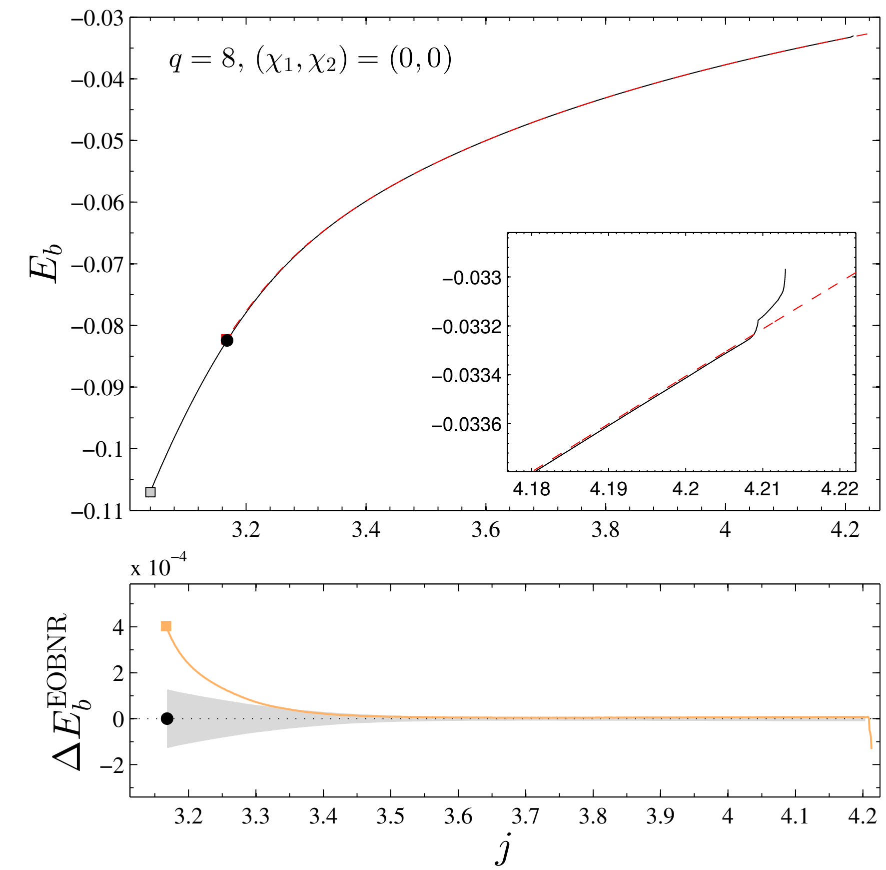
ENERGETICS - NONSPINNING

Binding energy vs angular momentum

(Llama NR data)

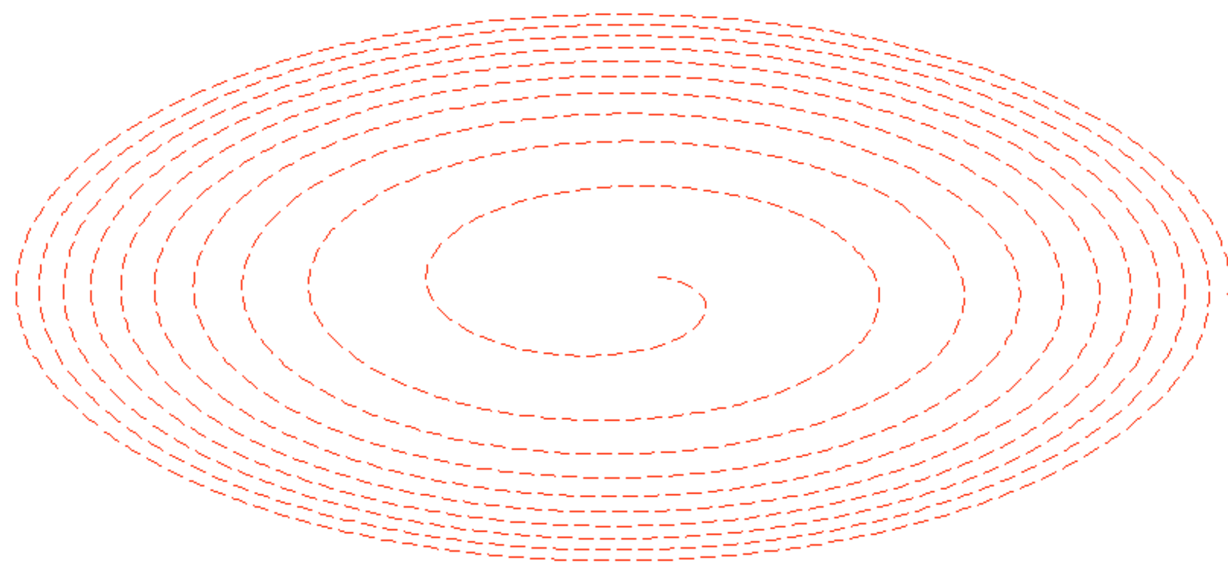
SXS data

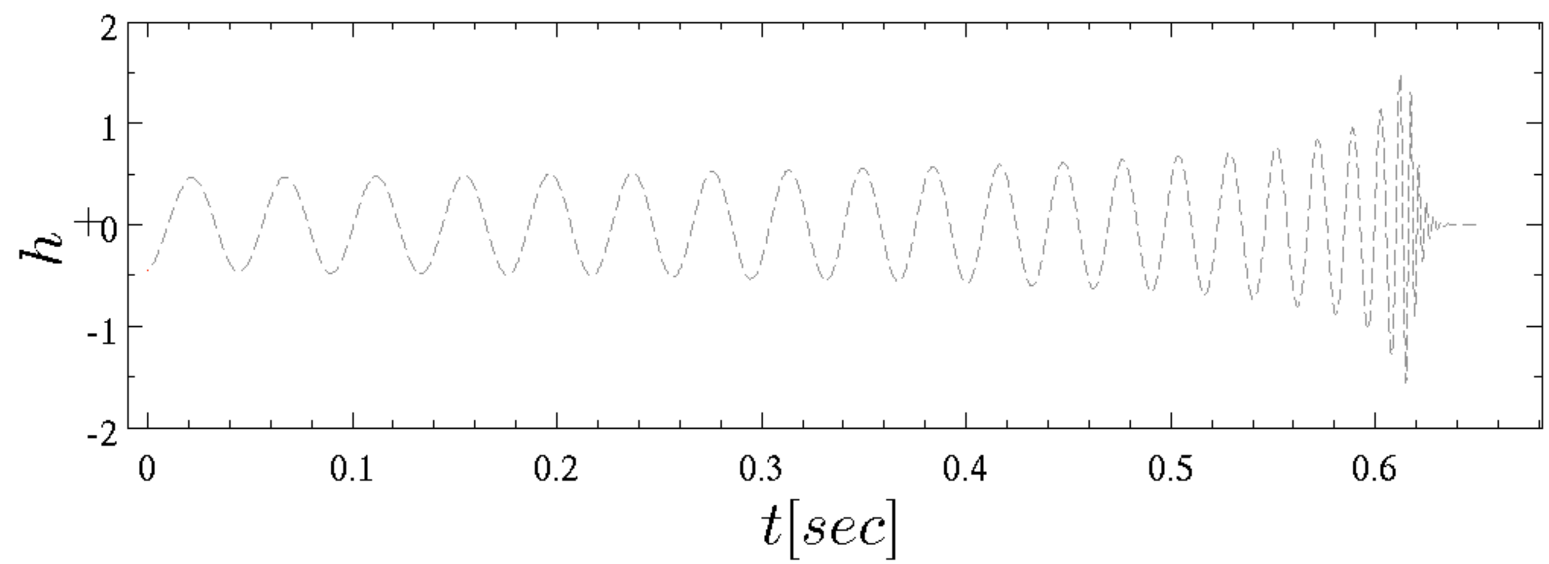




$$E_b = \frac{E - Mc^2}{\mu}$$

Nagar, Damour, Reisswig & Pollney, PRD 93 (2016), 044046





2
1
0
-1
-2
h+
0
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0.2
0.3
0.4
0.5
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t[sec]

EOB APPROACH TO THE DYNAMICS OF TWO SPINNING BLACK HOLES

Damour01, Buonanno-Chen-Damour06, Damour-Jaranowski-Schafer08, Barausse&Buonanno10,Nagar11,Barausse&Buonanno2011,Taracchini et al. 12, Balmelli&Jetzer2013, Pan et al. 2013

Nonspinning case: EOB description = deformation of test-particle Hamiltonian in a Schwarzschild background

Spinning case: EOB description = deformation of (spinning) test-particle Hamiltonian in a Kerr background

Deformation parameter:

$$\nu = \mu/M$$

Based on Hamiltonian formulation in the center of mass frame

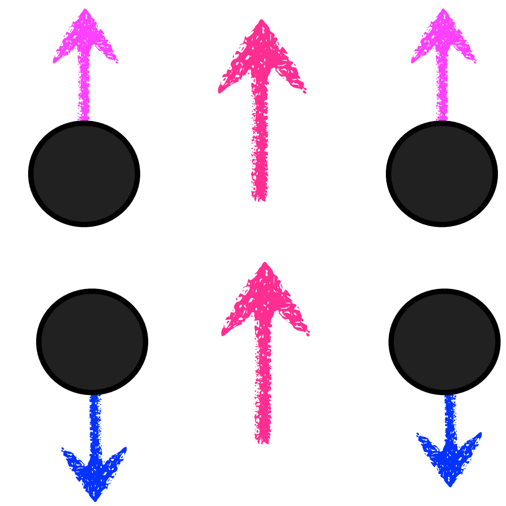
SPINNING BBHS

Spin-orbit & spin-spin couplings

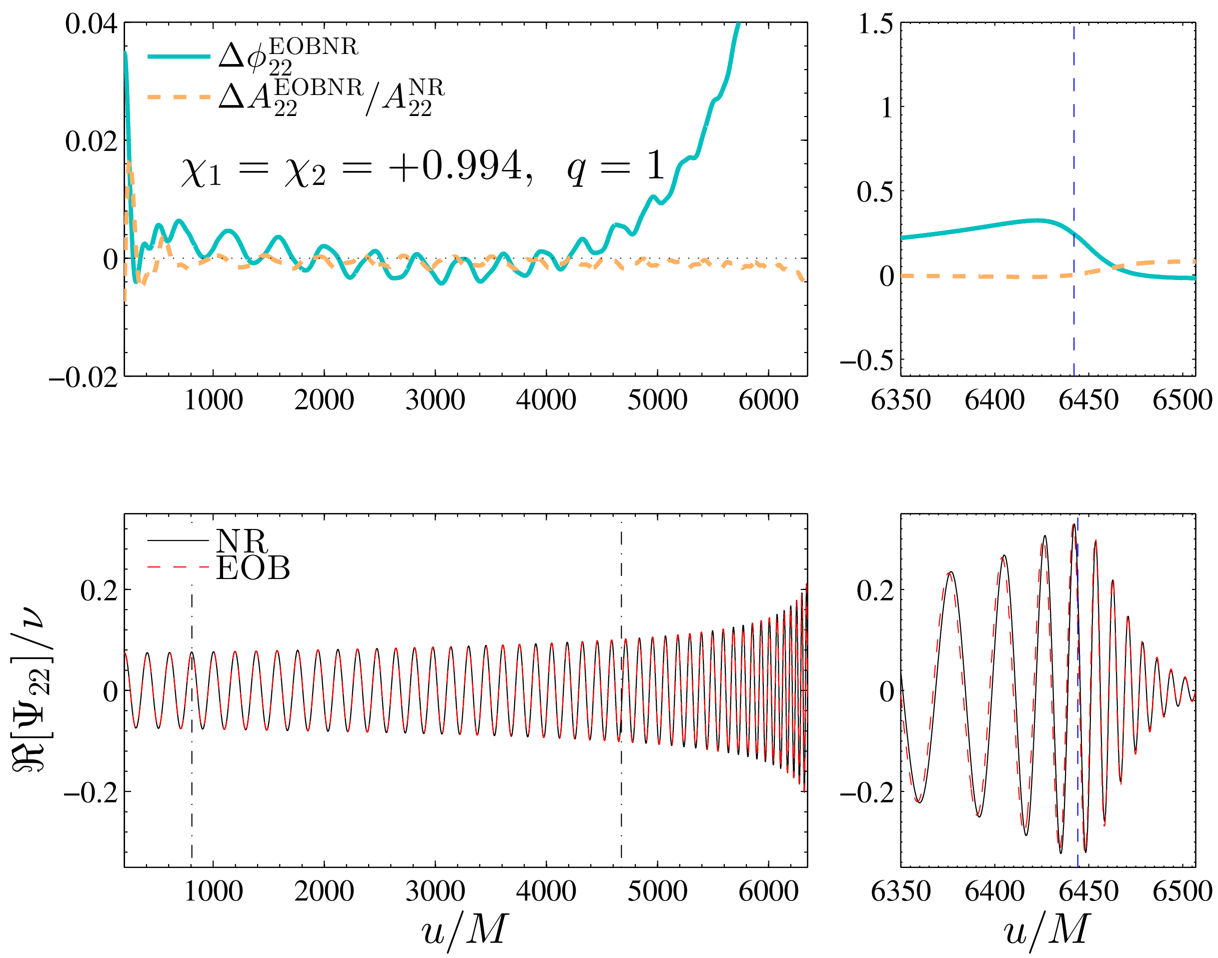
(i) Spins aligned with L: repulsive (slower) L-o-n-g-e-r INSPIRAL

(ii) Spins anti-aligned with L: attractive (faster) shorter INSPIRAL

(iii) Misaligned spins: precession of the orbital plane (waveform modulation)



$$\chi_{1,2} = \frac{c\,\mathbf{S}_{1,2}}{G m_{1,2}^2}$$



EOB/NR agreement: sophisticated (though rather simple) model for spin-aligned binaries

Damour&Nagar, PRD90 (2014), 024054 (Hamiltonian)

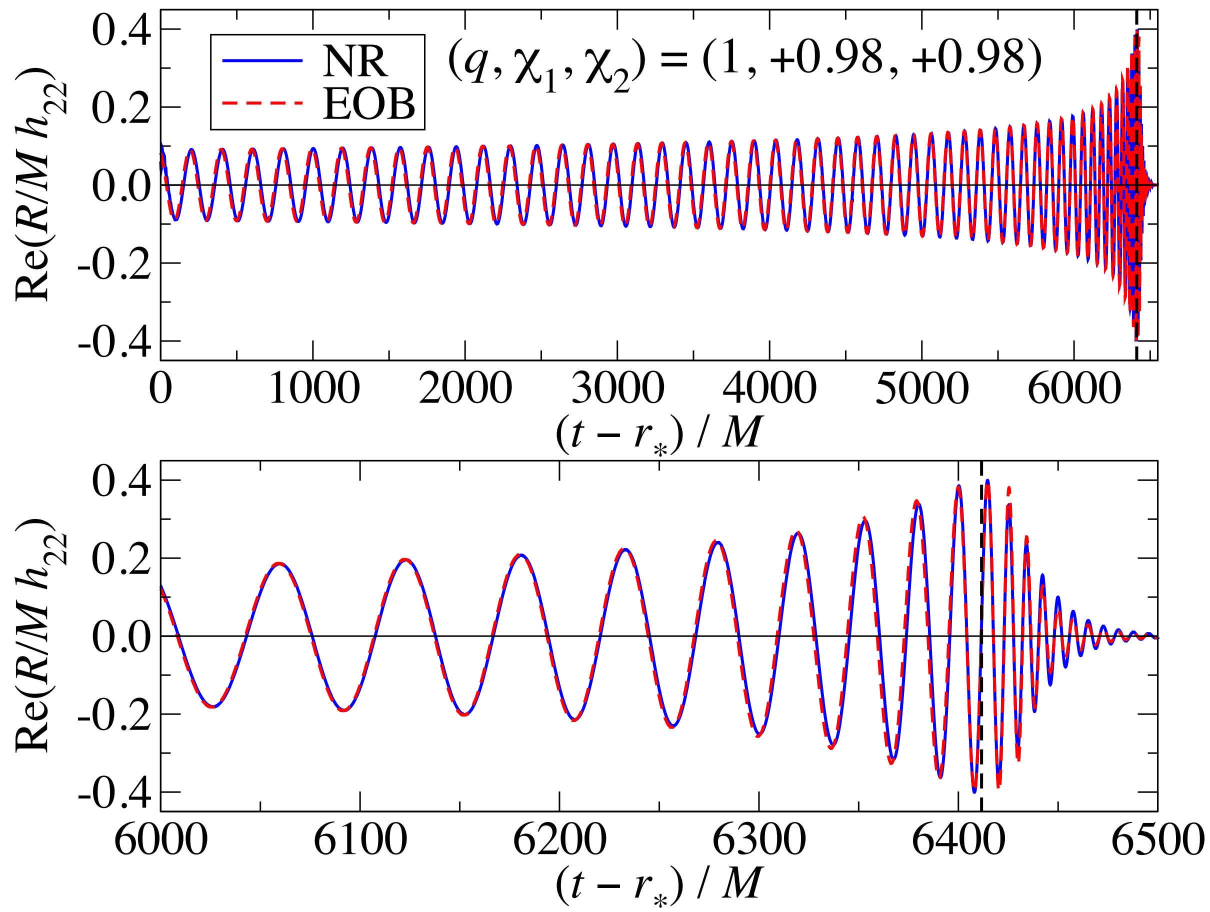
Damour&Nagar, PRD90 (2014), 044018 (Ringdown)

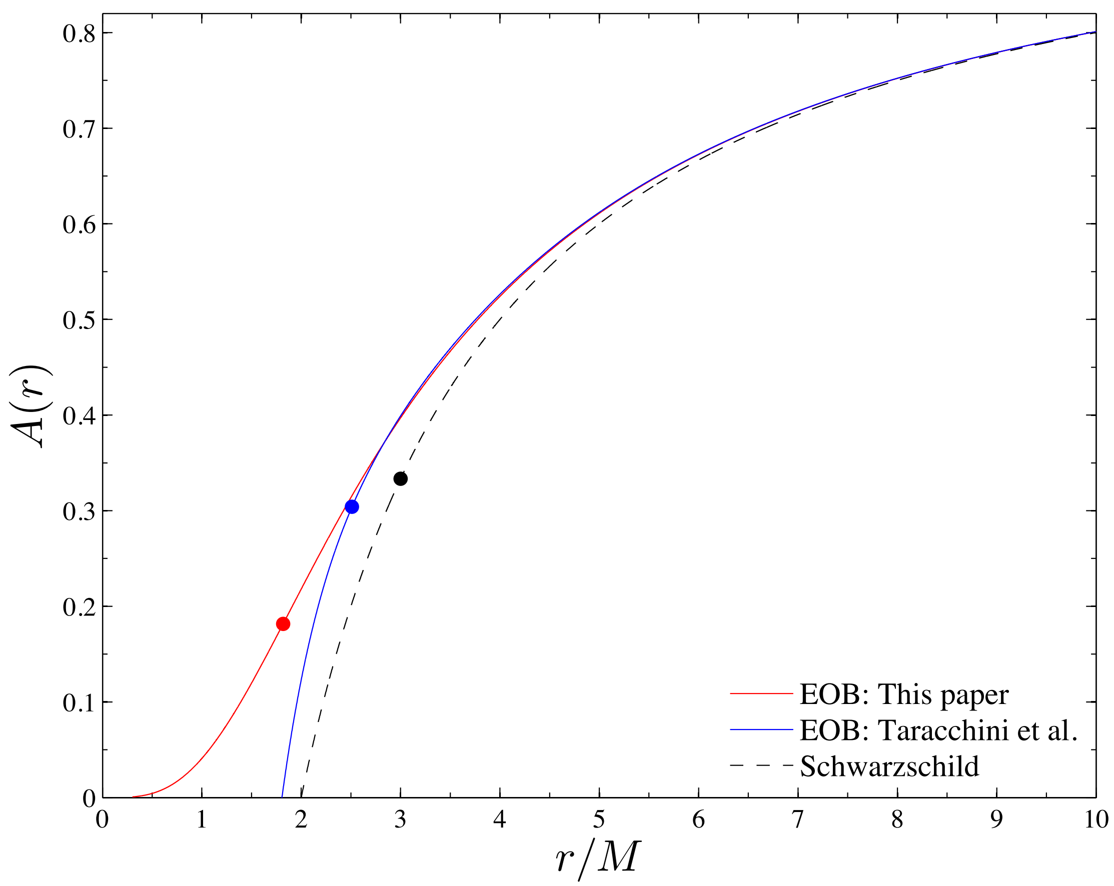
Nagar,Damour, Reisswig & Pollney, PRD 93 (2016), 044046

Calibrating a single, effective, 4.5PN (NNNLO) spin-orbit parameter

EOBNR MODEL USED FOR GW150914

Different EOB Hamiltonian [Barausse & Buonanno11, Taracchini et al.12]
SEOBNRv2: Taracchini, Buonanno et al., PRD 89, 061502 (R), 2014
SEOBNRv2_ROM_DoubleSpin: M. Puerrer, CQG 31, 195010 (2014)





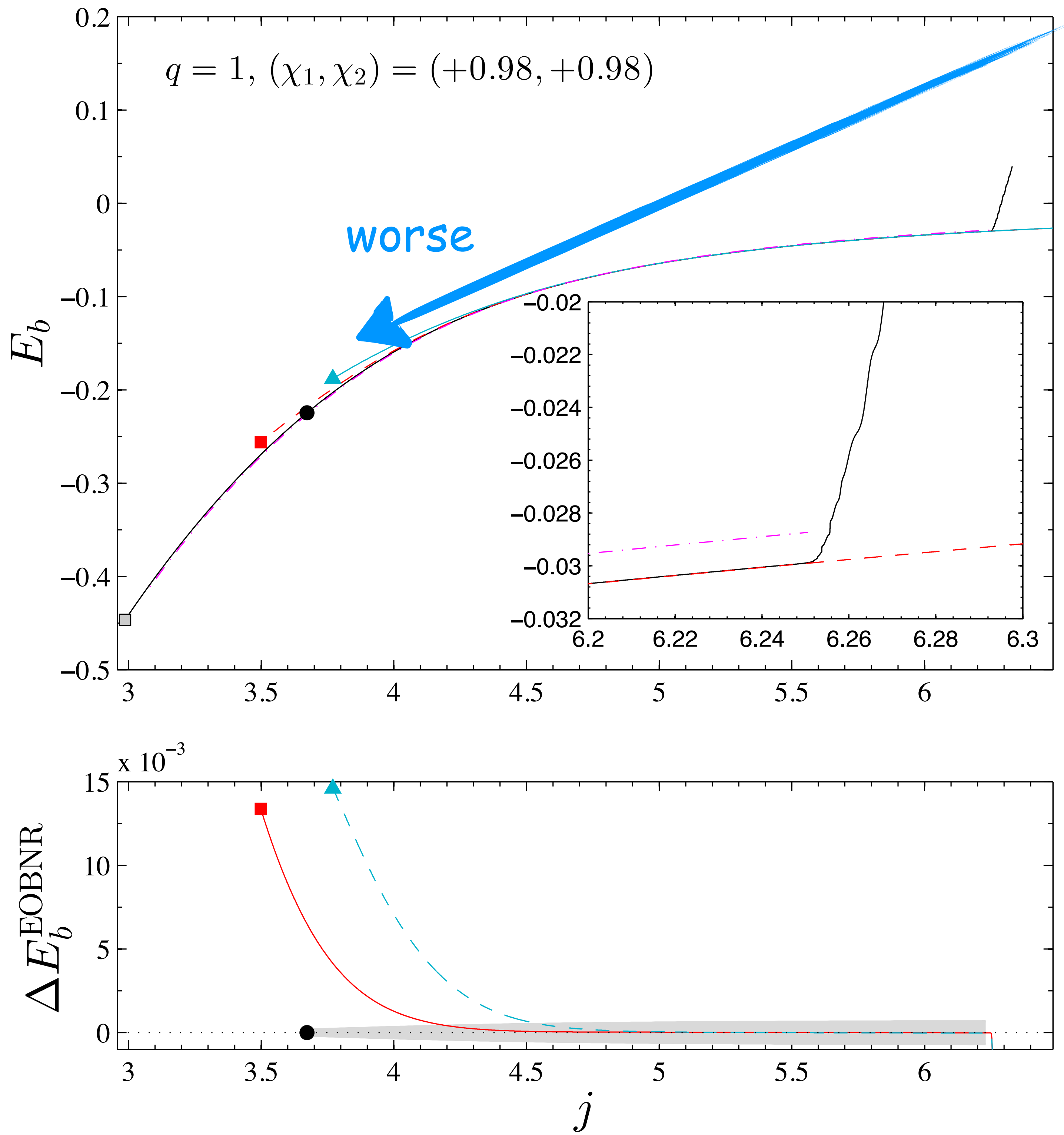
+ different spin-orbit & spin-spin sector

Effectively used to get the masses:
SEOBNRv2_ROM_DoubleSpin
IMRPhenom (Khan et al., 2015)

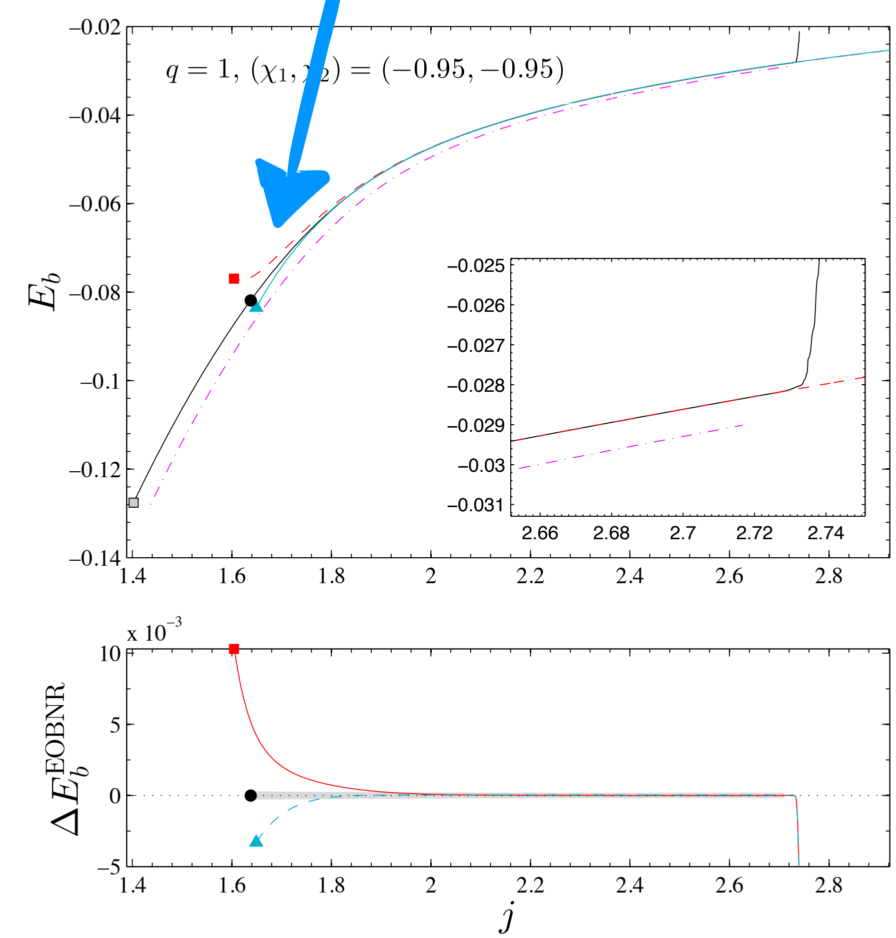
ENERGETICS

Taracchini, et al., 2014
SEOBNRv2 (LAL library)

worse



better



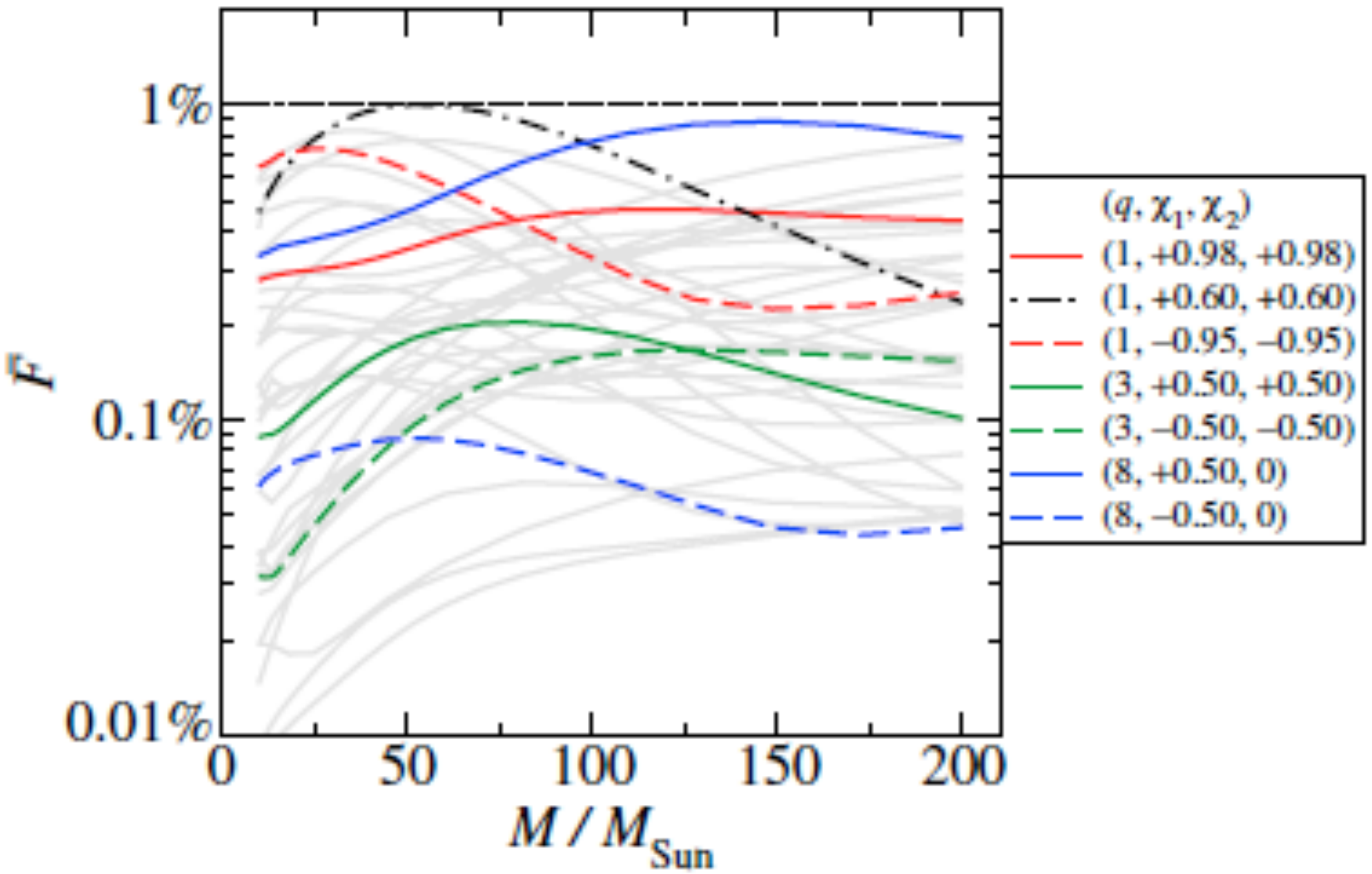
IHES EOBNR MODEL

SEOBNR_IHES model WAS NOT used for parameter estimation:
EOB/EOBNR UNFAITHFULNESS (40 NR SXS dataset)

$$\bar{F} \equiv 1 - \max_{t_0,\phi_0} \frac{\langle h_{22}^{\rm EOB}, h_{22}^{\rm NR} \rangle}{||h_{22}^{\rm EOB}||||h_{22}^{\rm NR}||}$$

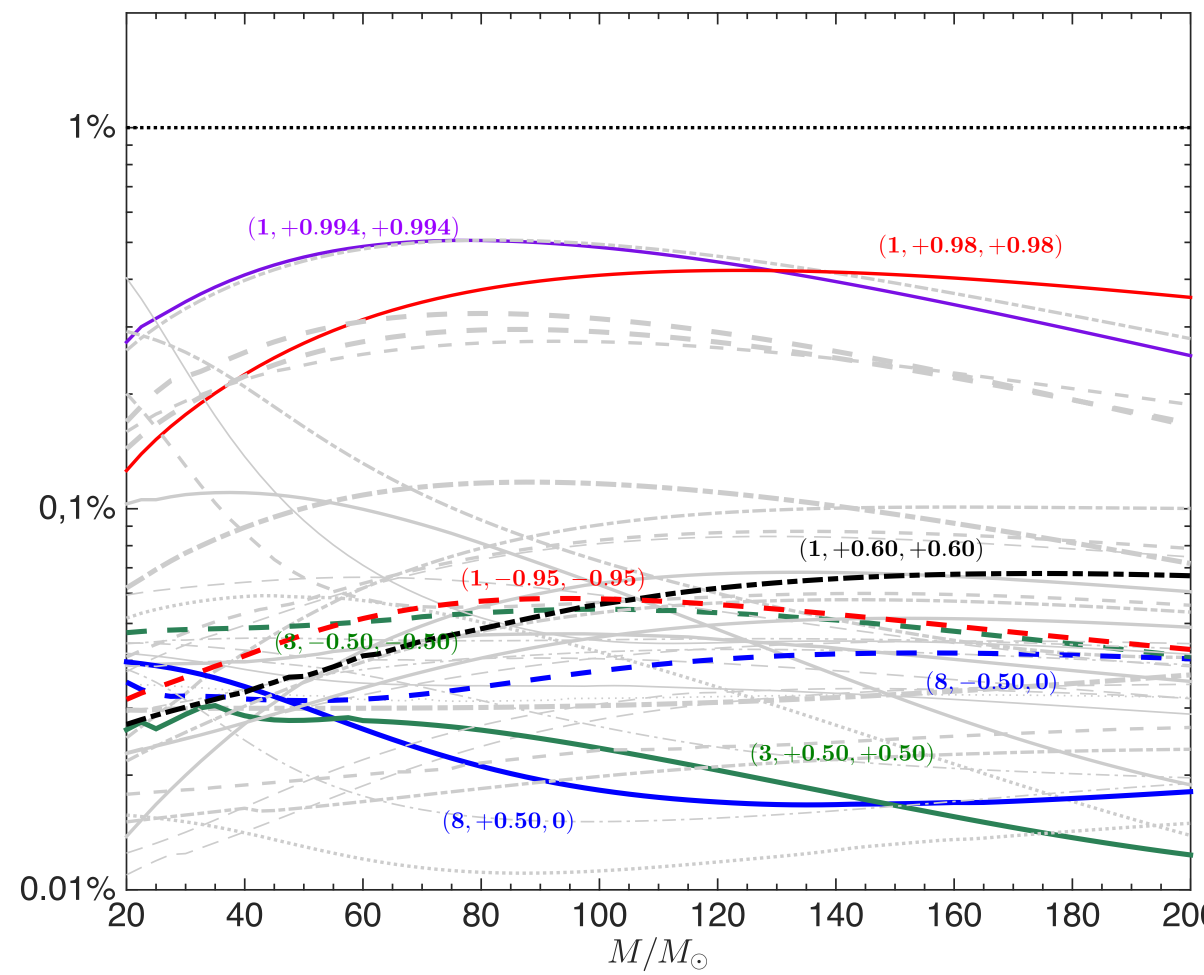
SEOBNRv2

Taracchini, Buonanno et al., PRD 89, 061502 (R), 2014

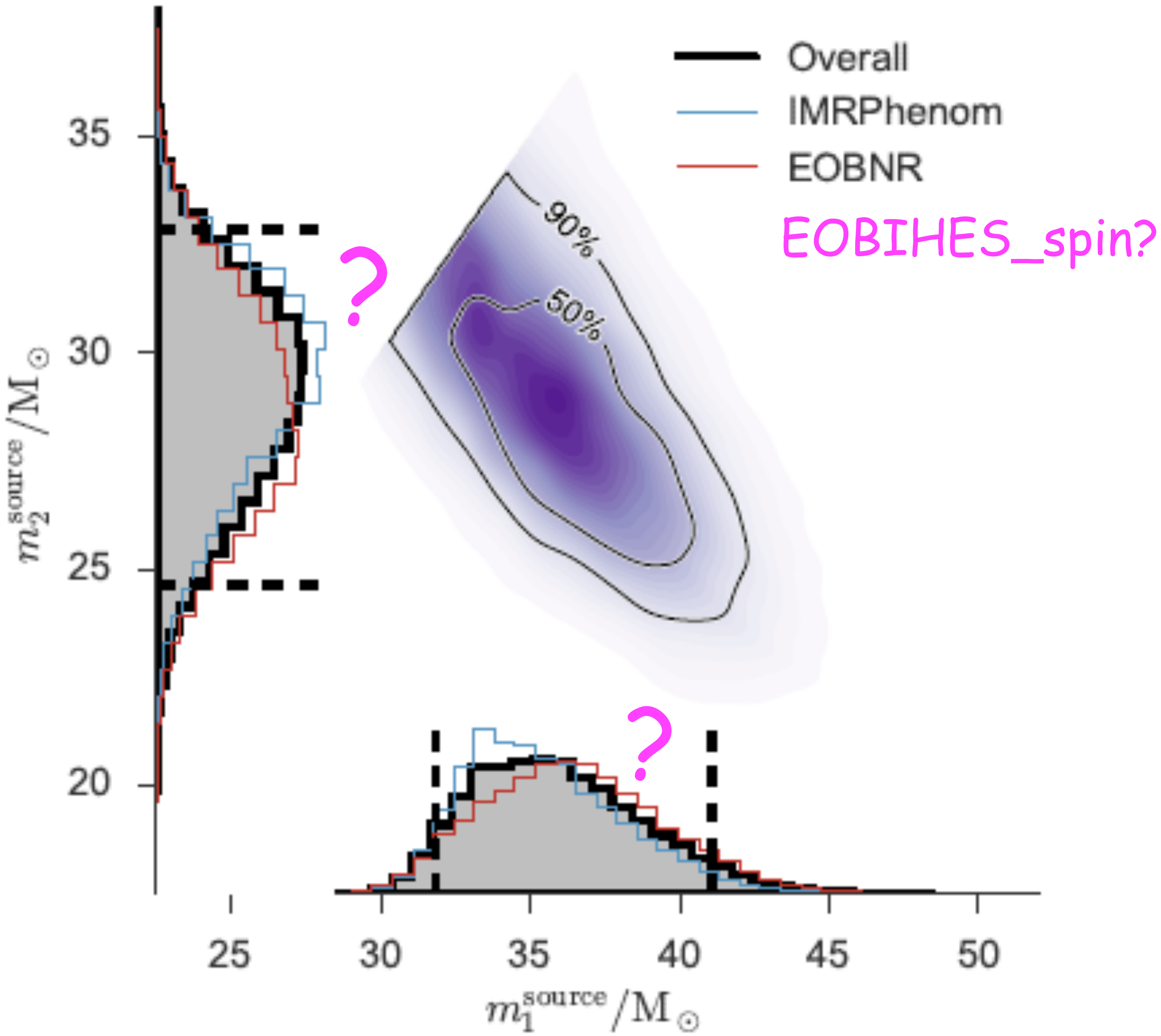


SEOB_ihes

Nagar, Damour, Reisswig & Pollney, PRD 93 (2016), 044046



IT WOULD BE INTERESTING TO KNOW ...



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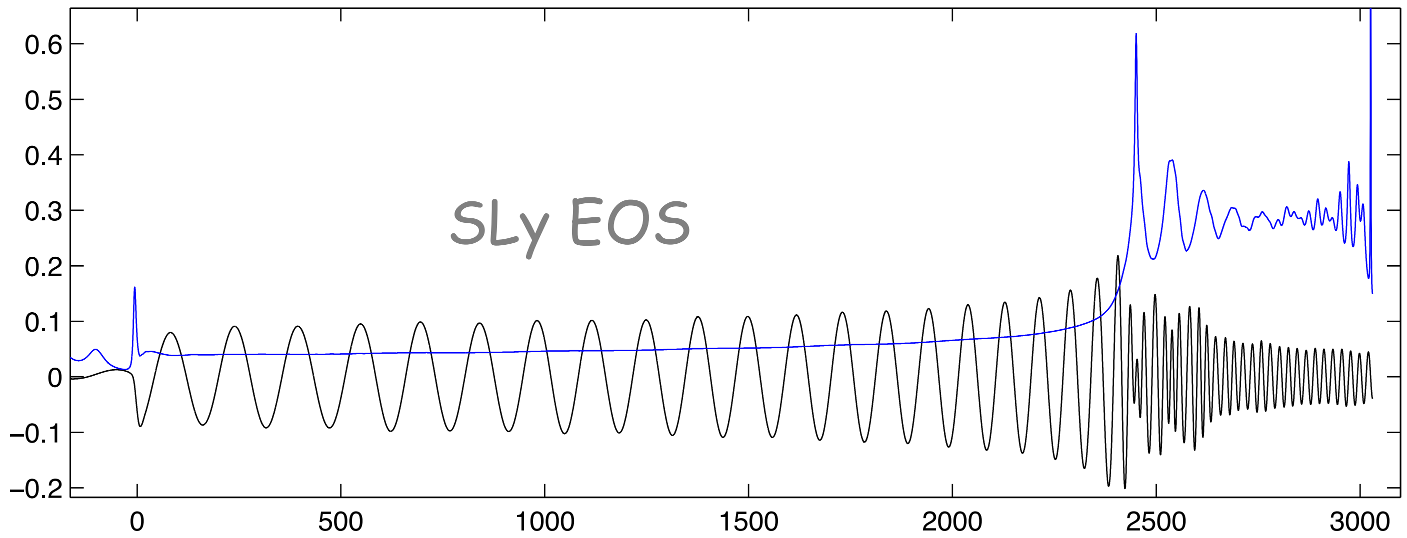
BINARY NEUTRON STARS (BNS)

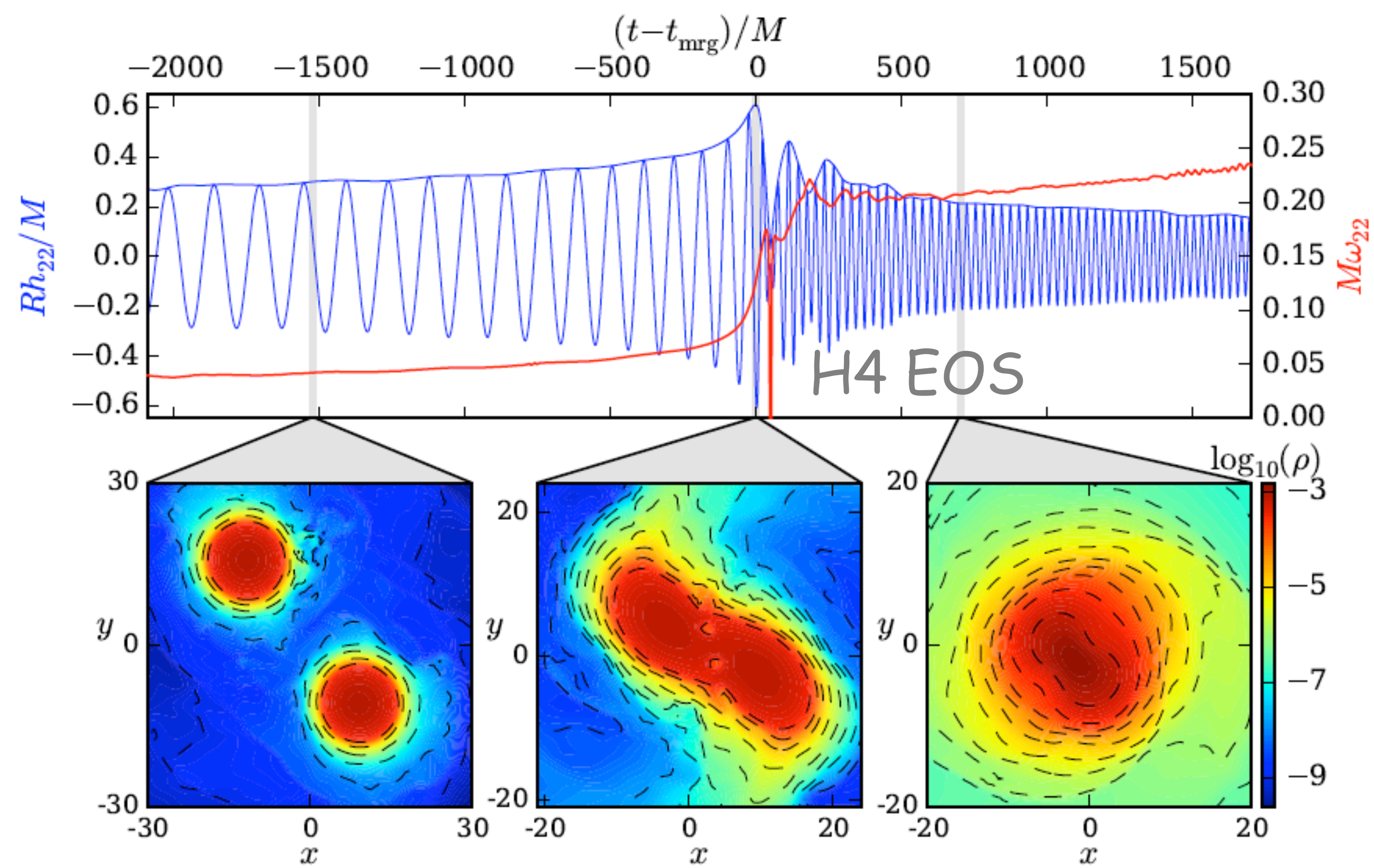


All BNS need is Love!

$$q = 1 \qquad M = 2.7 M_\odot$$

- Tidal effects
- Love numbers (tidal "polarization" constants)
- EOS dependence & "universality"

See:
Damour, 1983
Damour,Soffel,Xu, 1999-2001
Flanagan&Hinderer, PRD 2008
Damour&Nagar, PRD 2009
Damour&Nagar, PRD 2010
Damour,Nagar et al., PRL 2011
Bini,Damour&Faye, PRD2012
Bini&Damour, PRD 2014
Bernuzzi, Nagar, et al, PRL 2014
Bernuzzi, Nagar, Dietrich, PRL 2015
Bernuzzi, Nagar, Dietrich & Damour,PRL, 2015



THREE RESULTS

1. Numerical-relativity matches effective-one-body (EOB) analytical-relativity waveforms and dynamics essentially up to merger. Method to compute GW templates for LIGO/Virgo to measure EOS out of tidal effects

S. Bernuzzi, A. Nagar, T. Dietrich & T. Damour, PRL 114 (2015), 161103

"Modeling the Dynamics of Tidally Interacting Binary Neutron Stars up to Merger"

[Consistency with Hotokezaka et al., PRD 91 (2015) 6, 064060, notably with reduced eccentricity. With ourselves with improved simulations (unpublished) & Hinderer et al. 2016 (see AB talk)]

2. Quasi-universality in BNS merger (binding energy, angular momentum, GW frequency vs tidal coupling constant): explained using EOB theory

S. Bernuzzi, A. Nagar, S. Balmelli, T. Dietrich & M. Ujevic, PRL 112 (2014), 201101

"Quasiuniversal properties of neutron star mergers"

3. Quasi-universality of post-merger Mf_2 frequency vs tidal coupling constant

S. Bernuzzi, T. Dietrich & A. Nagar, PRL 115 (2015), 091101

"Towards a description of the complete gravitational wave spectrum of neutron star mergers"

Unifying description of inspiral, merger and post-merger phases

TIDAL EFFECTS IN EOB FORMALISM

Tidal extension of EOB formalism: nonminimal worldline couplings

$$\Delta S_{\text{nonminimal}} = \sum_A \frac{1}{4}\mu_2^A \int ds_A \left(u^\mu u^\nu R_{\mu\alpha\nu\beta}\right)^2 + \dots$$

Damour&Esposito-Farèse96, Goldberger&Rothstein06, TD&AN09

Relativistic Love number

Modifications of the EOB effective metric...

$$\begin{aligned} A(r) &= A_r^0 + A^{\text{tidal}}(r) \\ A^{\text{tidal}}(r) &= -\kappa_2^T u^6 \left(1 + \bar{\alpha}_1 u + \bar{\alpha}_2 u^2 + \dots\right) + \dots \end{aligned}$$

And tidal modifications of GW waveform & radiation reaction

- Need analytical theory for computing $\mu_2,\ \kappa_2^T,\ \bar{\alpha}_1 \dots$
- (?)Need accurate NR simulations to "calibrate" the higher-order PN tidal contributions, that may be quite important during the late inspiral

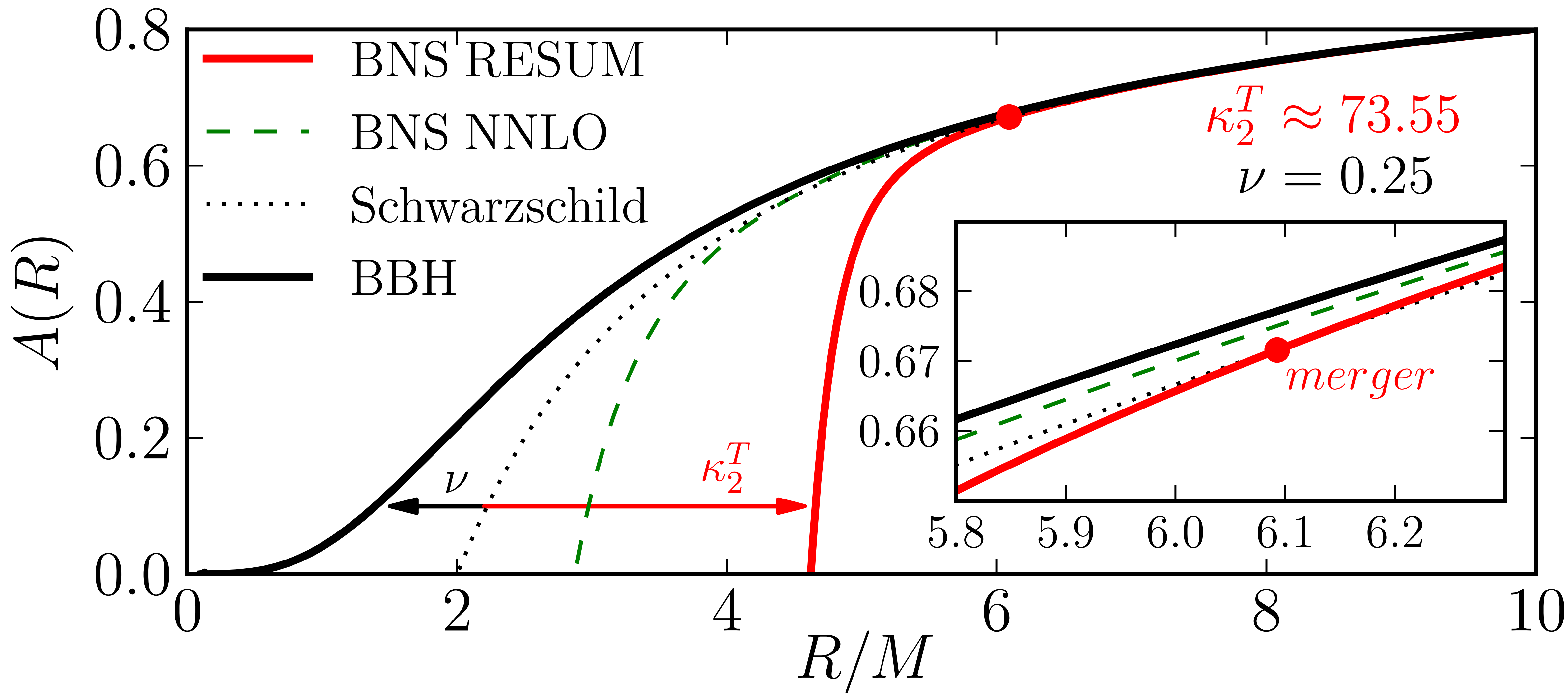
RESUMMED TIDAL INTERACTION

Bini&Damour (2015) resummed expression for $\hat{A}_\ell^{\rm tidal}$

Presence of a pole: potential strongly attractive @ mrg

$$A_T^{(+)}(u;\nu) \equiv -\sum_{\ell=2}^{4}\left[\kappa_A^{(\ell)} u^{2\ell+2} \hat{A}_A^{(\ell^+)} + (A \leftrightarrow B)\right]$$

$$\hat{A}_A^{(2^+)}(u) = 1 + \frac{3u^2}{1-r_{\rm LR}u} + \frac{X_A \tilde{A}_1^{(2^+)1\rm SF}}{(1-r_{\rm LR}u)^{7/2}} + \frac{X_A^2 \tilde{A}_2^{(2^+)2\rm SF}}{(1-r_{\rm LR}u)^p}$$



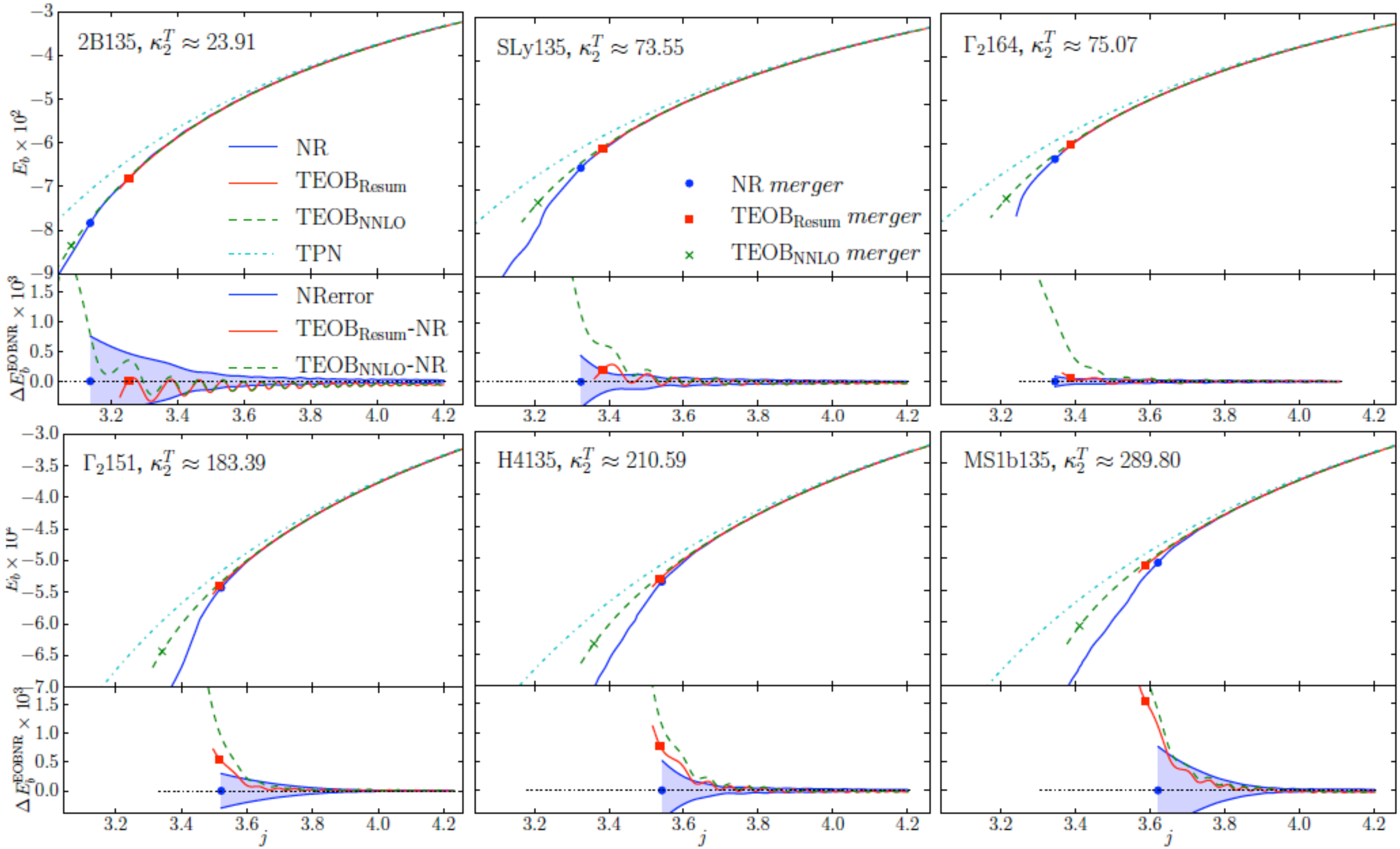


FIG. 2: Energetics: comparison between NR data, TEOB$_{\rm Resum}$, TEOB$_{\rm NNLO}$ and TPN. Each bottom panel shows the two EOB-NR differences. The filled circles locate the merger points (top) and the corresponding differences (bottom). The shaded area indicates the NR uncertainty. The TEOB$_{\rm Resum}$ model displays, globally, the smallest discrepancy with NR data (notably for merger quantities), supporting the theoretical, light-ring driven, amplification of the relativistic tidal factor.

S. Bernuzzi, A. Nagar, T. Dietrich & T. Damour, PRL 114 (2015), 161103

Waveform

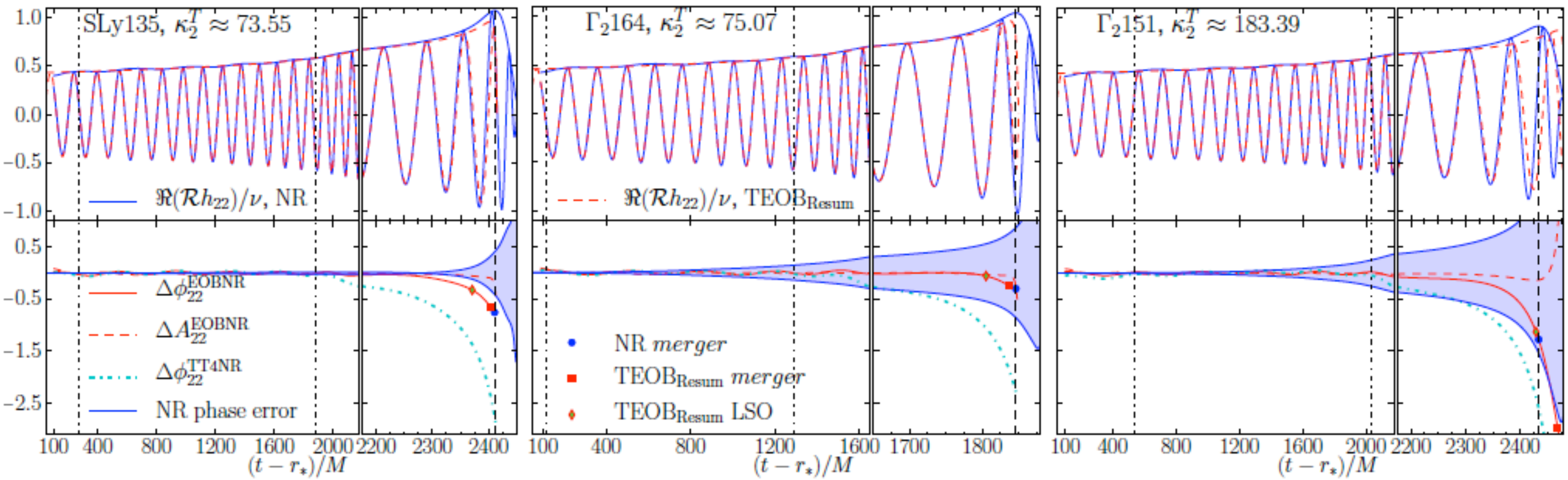


FIG. 3: Phasing and amplitude comparison (versus NR retarded time) between $\mathrm{TEOB}_{\mathrm{Resum}}$, NR and the phasing of TT4 for three representative models. Waves are aligned on a time window (vertical dot-dashed lines) corresponding to $I_\omega \approx (0.04, 0.06)$. The markers in the bottom panels indicate: the crossing of the $\mathrm{TEOB}_{\mathrm{Resum}}$ LSO radius; NR (also with a dashed vertical line) and EOB merger moments.

Name	EOS	κ_2^T	r_{LR}	$\mathcal{C}_{A,B}$	$M_{A,B}[M_\odot]$	$M^0_{\mathrm{ADM}}[M_\odot]$	$\mathcal{J}^0_{\mathrm{ADM}}[M_\odot^2]$	$\Delta\phi^{\mathrm{TT4}}_{\mathrm{NRmrg}}$	$\Delta\phi^{\mathrm{TEOB_{NNLO}}}_{\mathrm{NRmrg}}$	$\Delta\phi^{\mathrm{TEOB_{Resum}}}_{\mathrm{NRmrg}}$	$\delta\phi^{\mathrm{NR}}_{\mathrm{NRmrg}}$
2B135	2B	23.9121	3.253	0.2049	1.34997	2.67762	7.66256	-1.25	-0.19	$+0.57^a$	± 4.20
SLy135	SLy	73.5450	3.701	0.17381	1.35000	2.67760	7.65780	-2.75	-1.79	-0.75	± 0.40
$\Gamma_2 164$	$\Gamma = 2$	75.0671	3.728	0.15999	1.64388	3.25902	11.11313	-2.29	-1.36	-0.31	± 0.90
$\Gamma_2 151$	$\Gamma = 2$	183.3911	4.160	0.13999	1.51484	3.00497	9.71561	-2.60	-1.92	-1.27	± 1.20
H4135	H4	210.5866	4.211	0.14710	1.35003	2.67768	7.66315	-3.02	-2.43	-1.88	± 1.04
MS1b135	MS1b	289.8034	4.381	0.14218	1.35001	2.67769	7.66517	-3.25	-2.84	-2.45	± 3.01

SEOB_IHES

Nonspinning BBHs & BNS (tides)

Free download Matlab code: https://eob.ihes.fr.
(2,1) & (3,3) modes included

Spinning (nonprecessing) BBHs:
Matlab (development version)
C++ version (Philipp Fleig), including tides.

Some (early) performance numbers for equal-mass, nonspinning:

$M f_0 = 2 \times 10^{-4}$ 628s $(r_0 = 120M)$

$M f_0 = 1 \times 10^{-4}$ 6619s $(r_0 = 216M)$

MacBook pro, Intel Core i7, 2.7GHz

CONCLUSIONS

SEOB_ihes: Alternative model to SEOBNRv2 for spin-aligned BBHs.

Different theoretical elements and different calibration than SEOBNRv2.

Performances in parameter estimations should be explored/compared.

Careful EOB/NR comparisons of both waveforms & energetics (including BNS)

Matlab code free available. C++ code available on request

The wave(s) have passed....