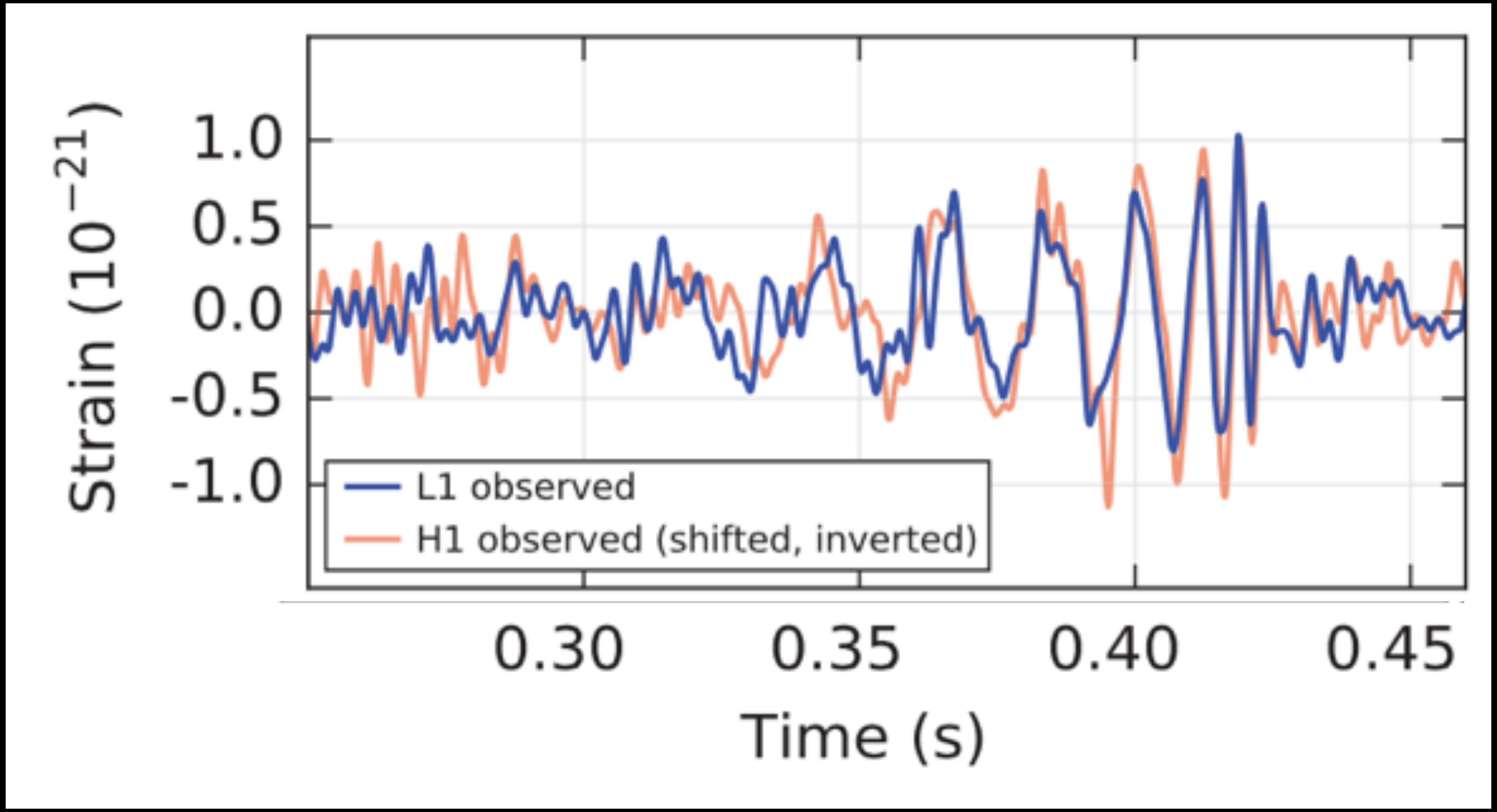
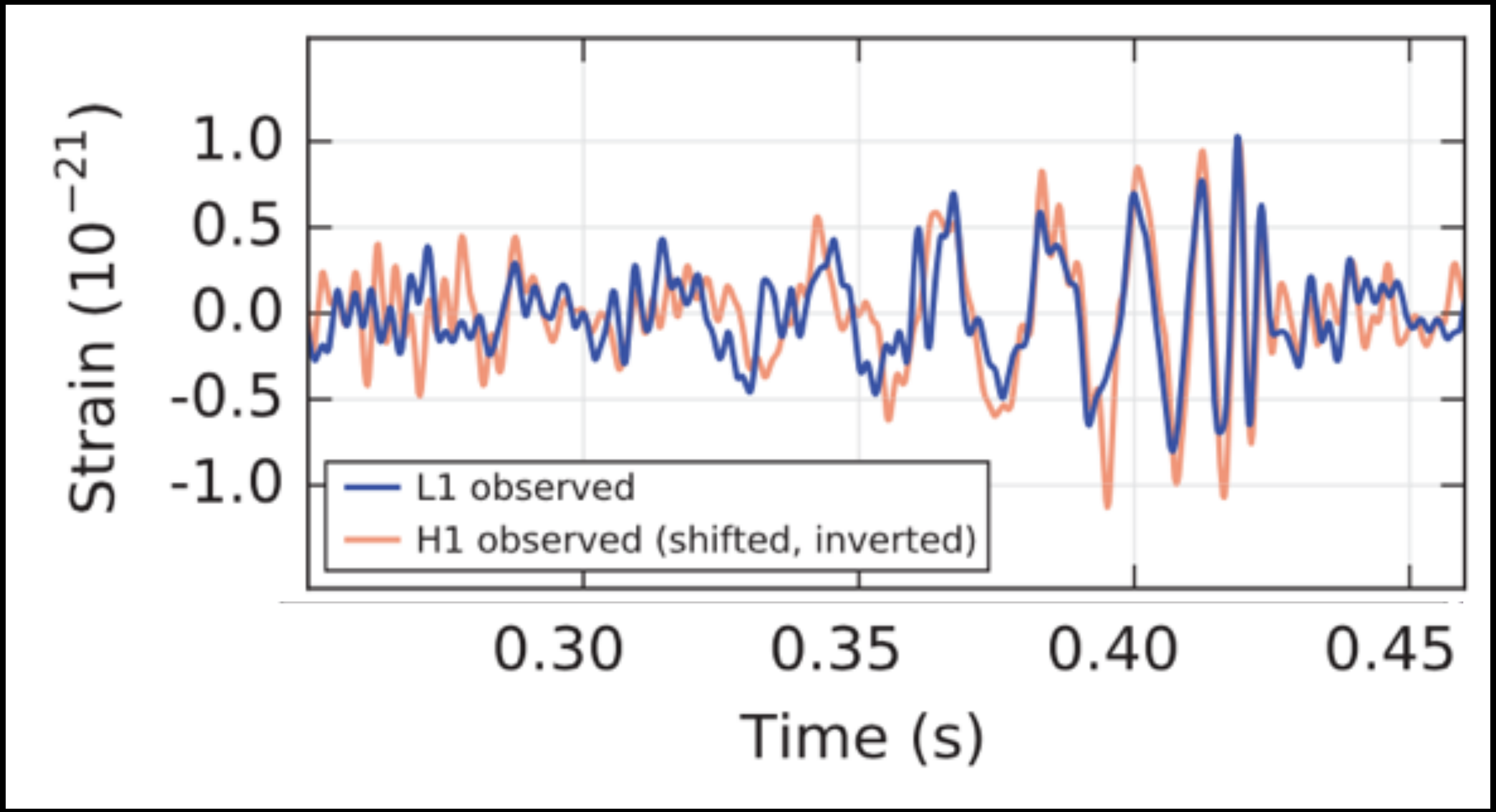
Waveform modelling with numerical relativity: the key to decode GW150914

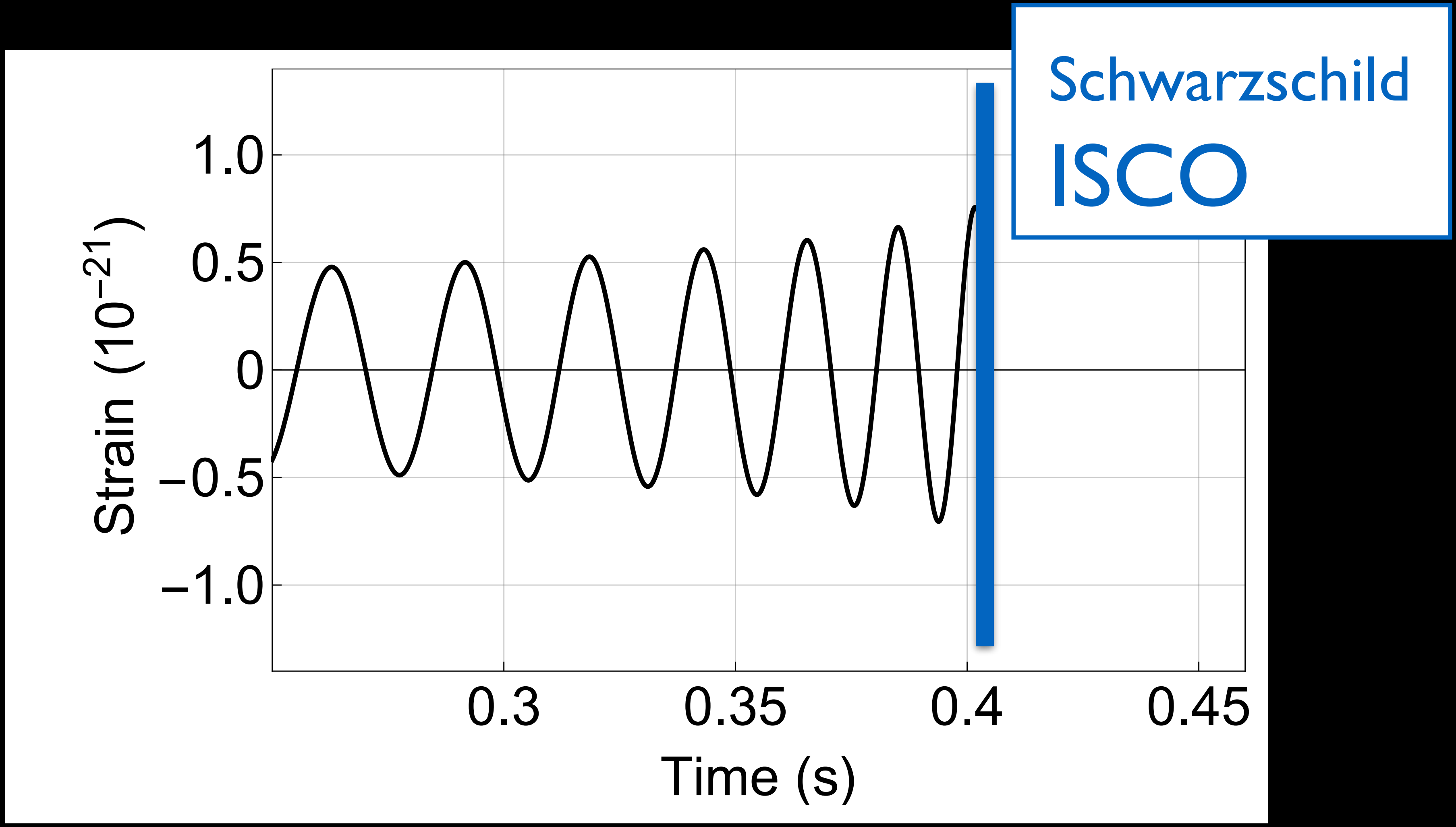
Mark Hannam
Cardiff University

GW150914 meeting
Hannover, May 24 2016



Strain (10^{-21})
1.0
0.5
0.0
-0.5
-1.0
L1 observed
H1 observed (shifted, inverted)
0.30
0.35
0.40
0.45
Time (s)

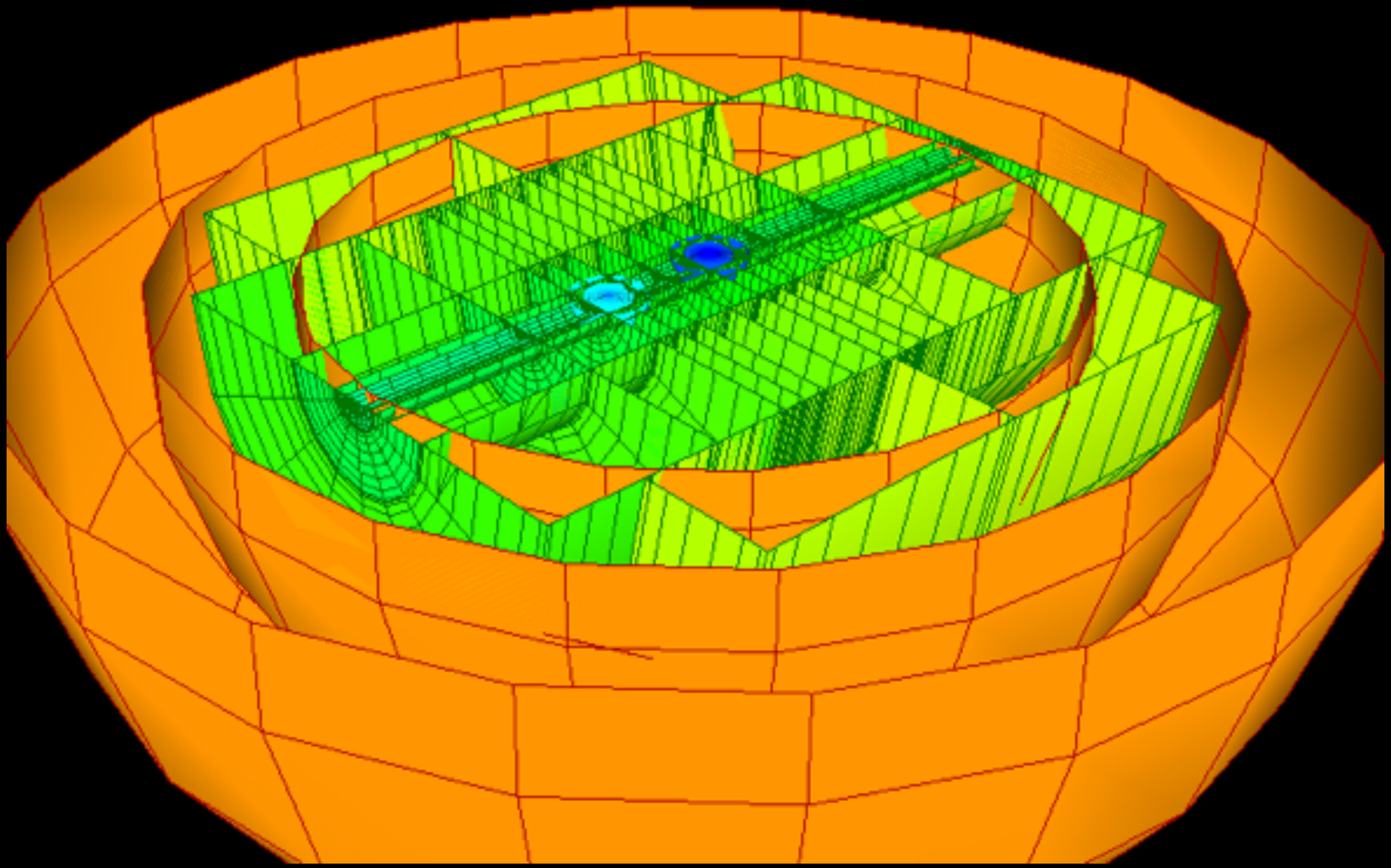

1.0
0.5
0.0
-0.5
-1.0
Strain (10^{-21})
L1 observed
H1 observed (shifted, inverted)
0.30
0.35
0.40
0.45
Time (s)


1.0
0.5
0
−0.5
−1.0
Strain (10^{-21})
0.3
0.35
0.4
0.45
Time (s)
Schwarzschild
ISCO

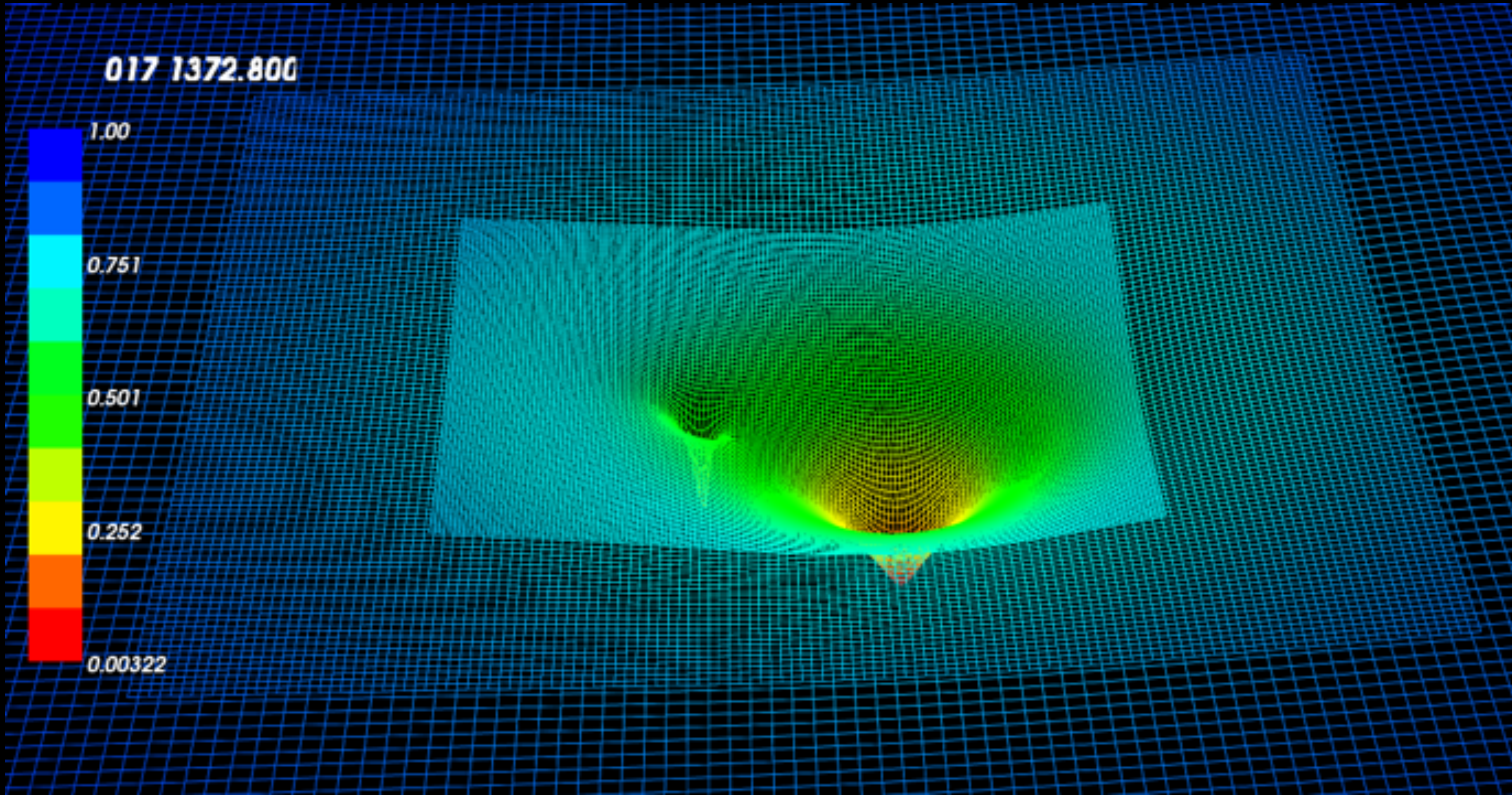
The binary black hole problem (1964—2005)

Boosted Three-Dimensional Black-Hole Evolutions with Singularity Excision

The accurate computational modeling of black-hole interactions is essential to the confident detection of astrophysical gravitational radiation by future space-based detectors such as LISA and by the LIGO/VIRGO/GEO complex of ground-based detectors currently under construction. The sensitivity of these detectors will be significantly enhanced if accurate computer simulations of black-hole mergers can produce predictions of radiation waveforms [1]. The Binary Black Hole Grand Challenge Alliance [2] was funded in September 1993 to develop the computational infrastructure for accurate simulations of the coalescence of black-hole binaries. The primary objective of the resulting code will be the prediction of waveforms from binary black-hole mergers. In this Letter we report on an important step towards achieving such simulations.



Computational Cost

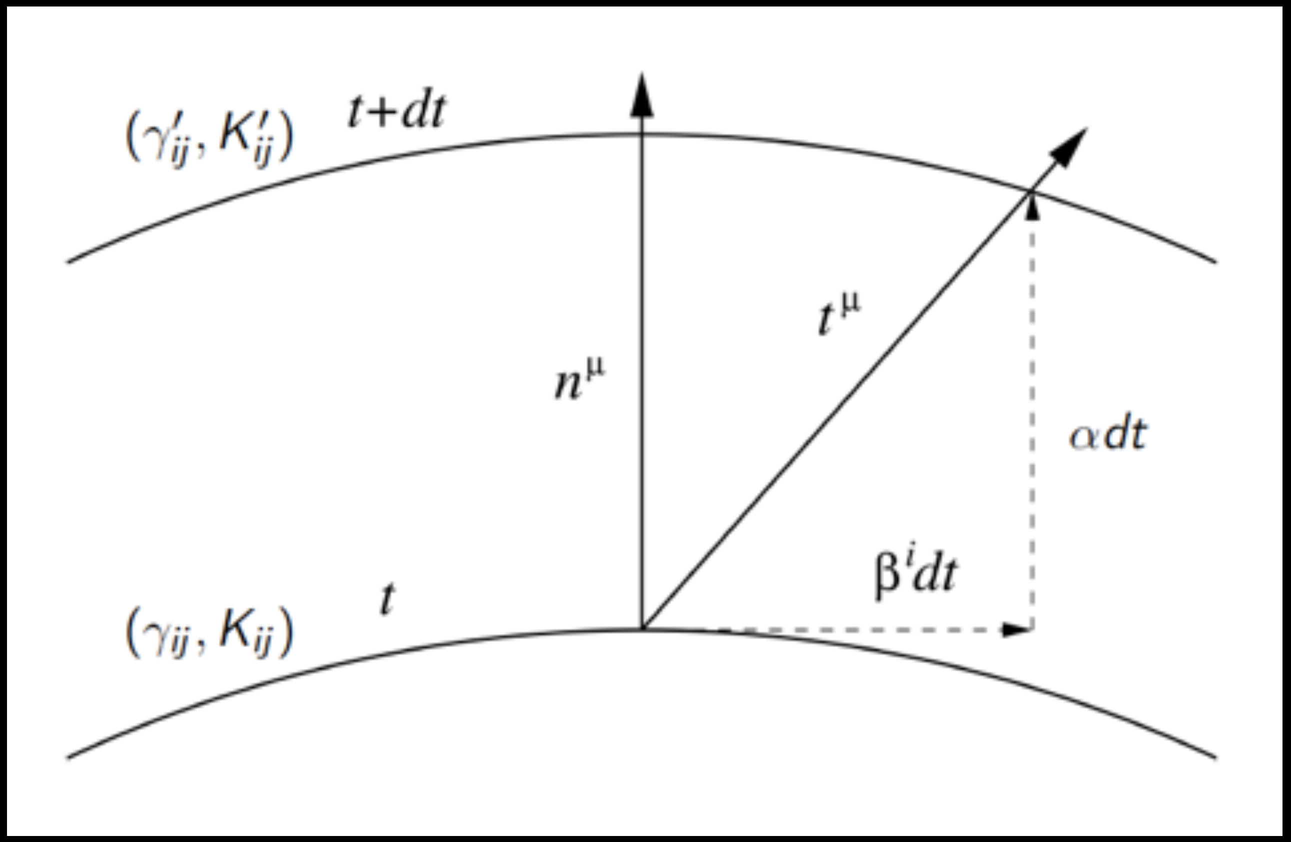


$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

Numerically stable formulations

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

(3+1 split)



$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

(3+1 split)

$$\begin{aligned} \bar{R} + K^2 - K_{ij}K^{ij} &= 0, \\ \bar{\nabla}_j K^{ij} - \gamma^{ij}\bar{\nabla}_j K &= 0. \end{aligned}$$

$$\begin{aligned} \partial_t \gamma_{ij} &= -2NK_{ij} + \bar{\nabla}_i \beta_j + \bar{\nabla}_j \beta_i, \\ \partial_t K_{ij} &= -\bar{\nabla}_i \bar{\nabla}_j N + N\left(R_{ij} - 2K_{ik}K^k_j + KK_{ij}\right) \\ &\quad + \beta^k \bar{\nabla}_k K_{ij} + K_{ik}\bar{\nabla}_j \beta^k + K_{kj}\bar{\nabla}_i \beta^k. \end{aligned}$$

[ADM (1962), York (1979)]

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

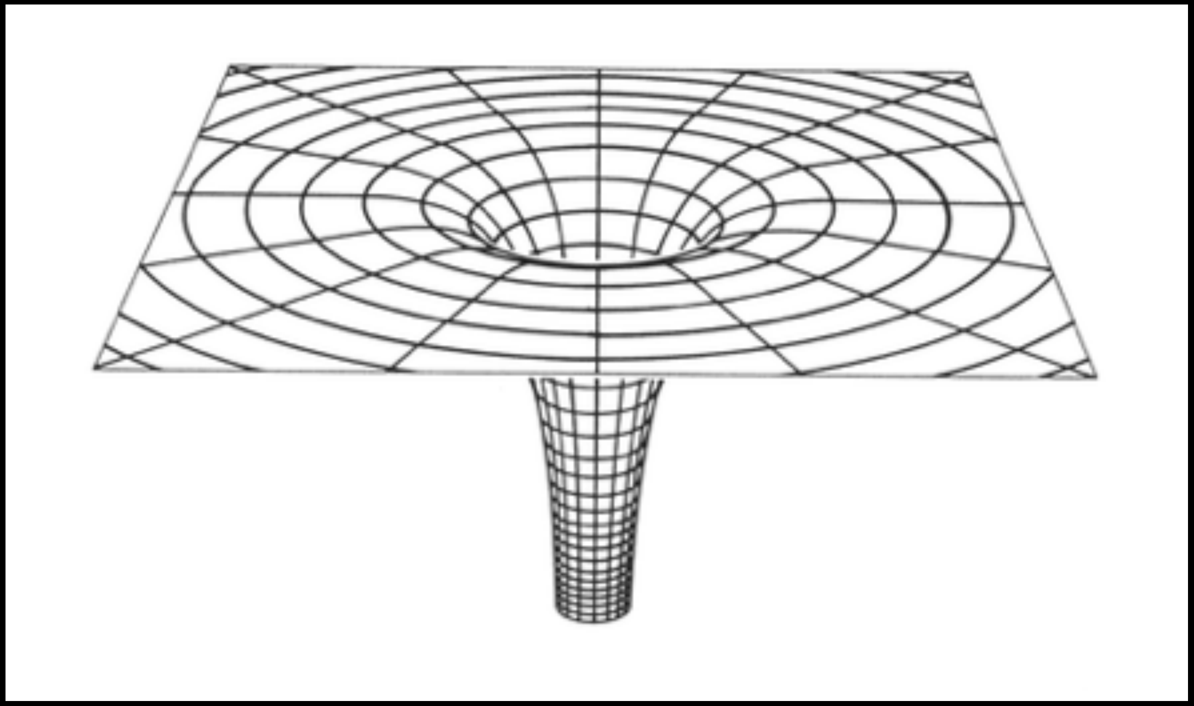
(3+1 split)

$$\begin{aligned} \bar{R} + K^2 - K_{ij}K^{ij} &= 0, \\ \bar{\nabla}_j K^{ij} - \gamma^{ij}\bar{\nabla}_j K &= 0. \end{aligned}$$

$$\begin{aligned} \partial_t \gamma_{ij} &= -2NK_{ij} + \bar{\nabla}_i \beta_j + \bar{\nabla}_j \beta_i, \\ \partial_t K_{ij} &= -\bar{\nabla}_i \bar{\nabla}_j N + N\left(R_{ij} - 2K_{ik}K_j^k + KK_{ij}\right) \\ &\quad + \beta^k \bar{\nabla}_k K_{ij} + K_{ik}\bar{\nabla}_j \beta^k + K_{kj}\bar{\nabla}_i \beta^k. \end{aligned}$$

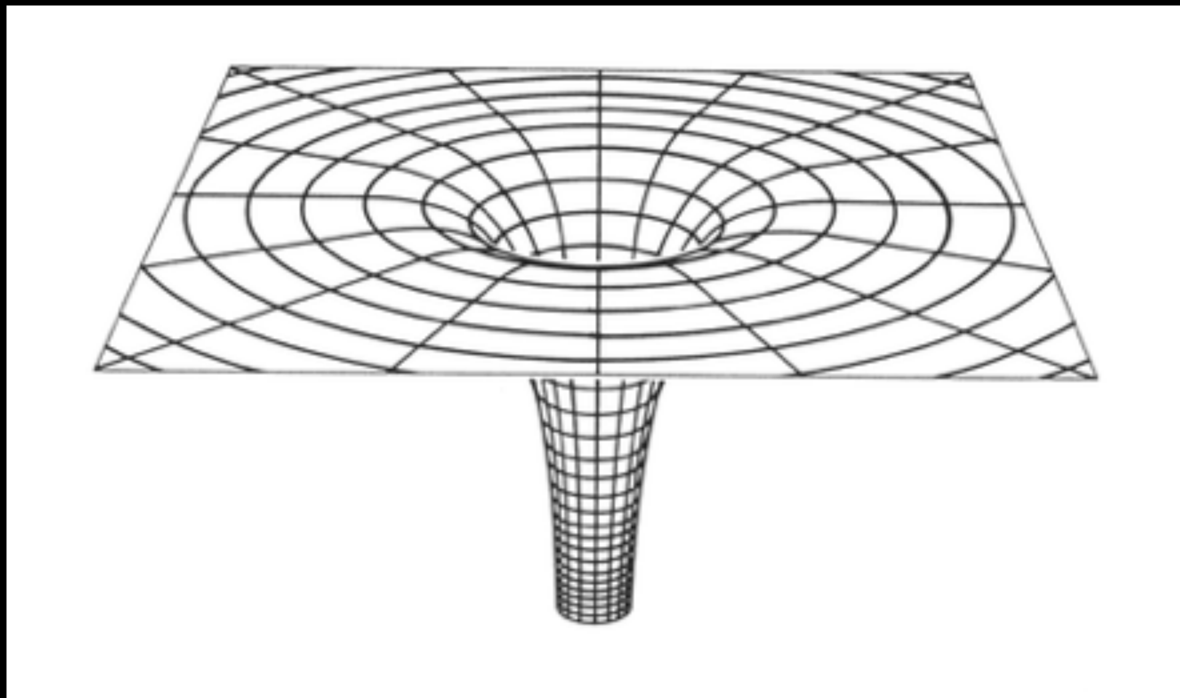
[ADM (1962), York (1979)]

? ? ?

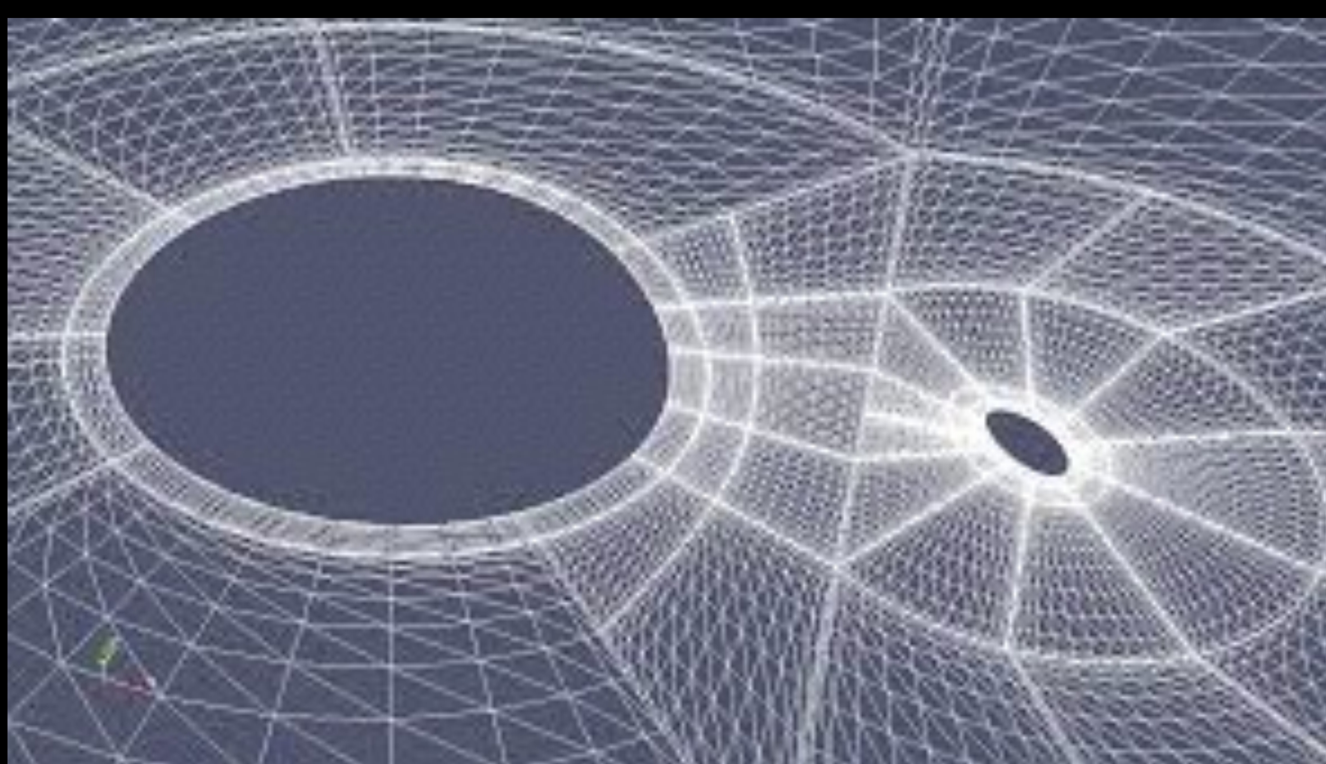


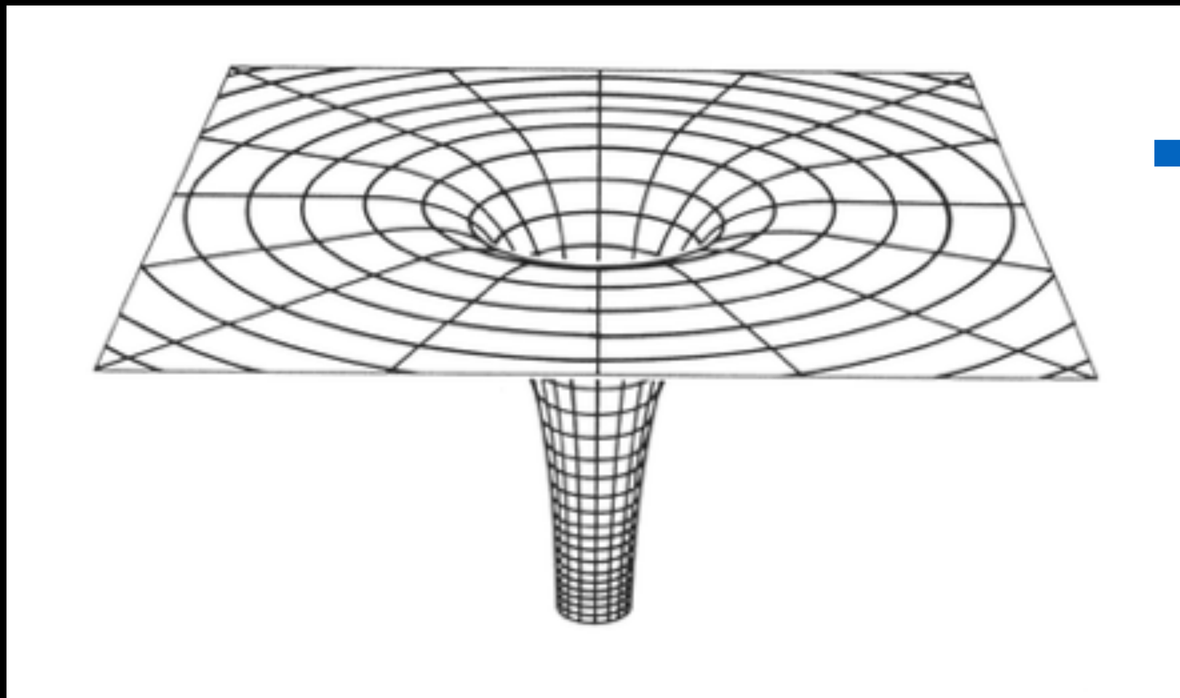
Black-hole singularities

Black-hole singularities

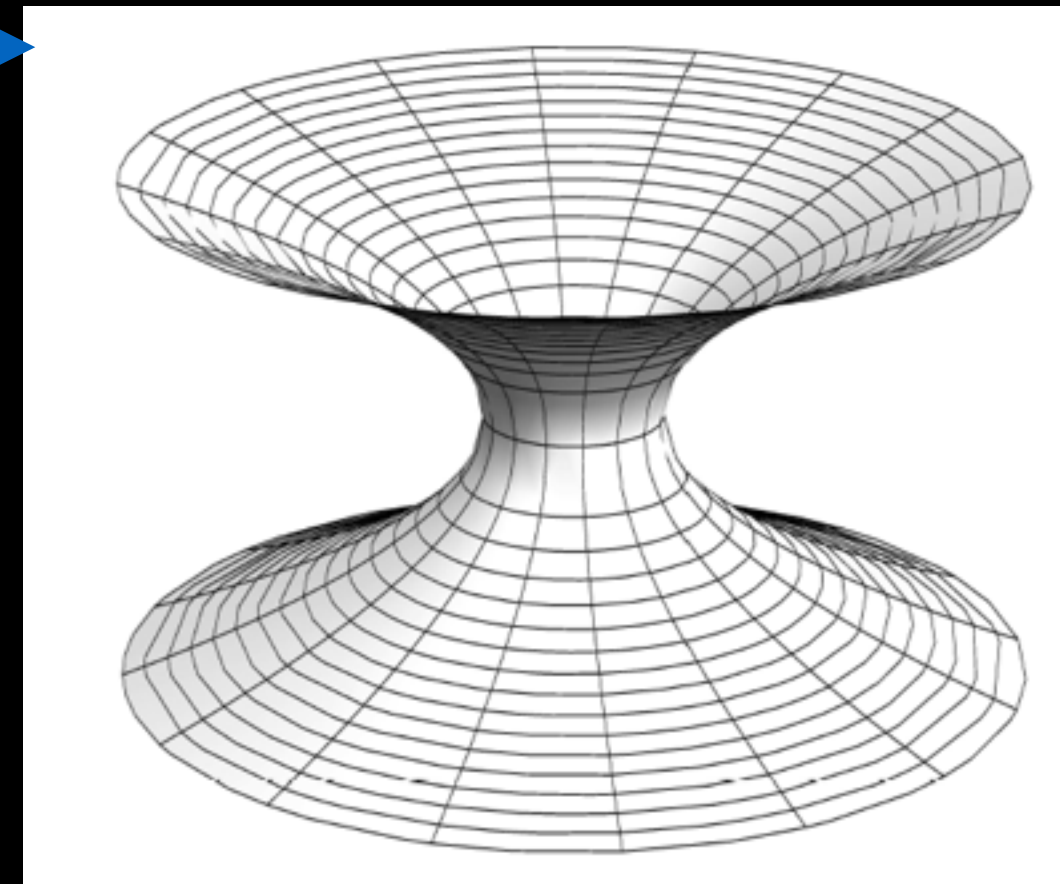


Excision

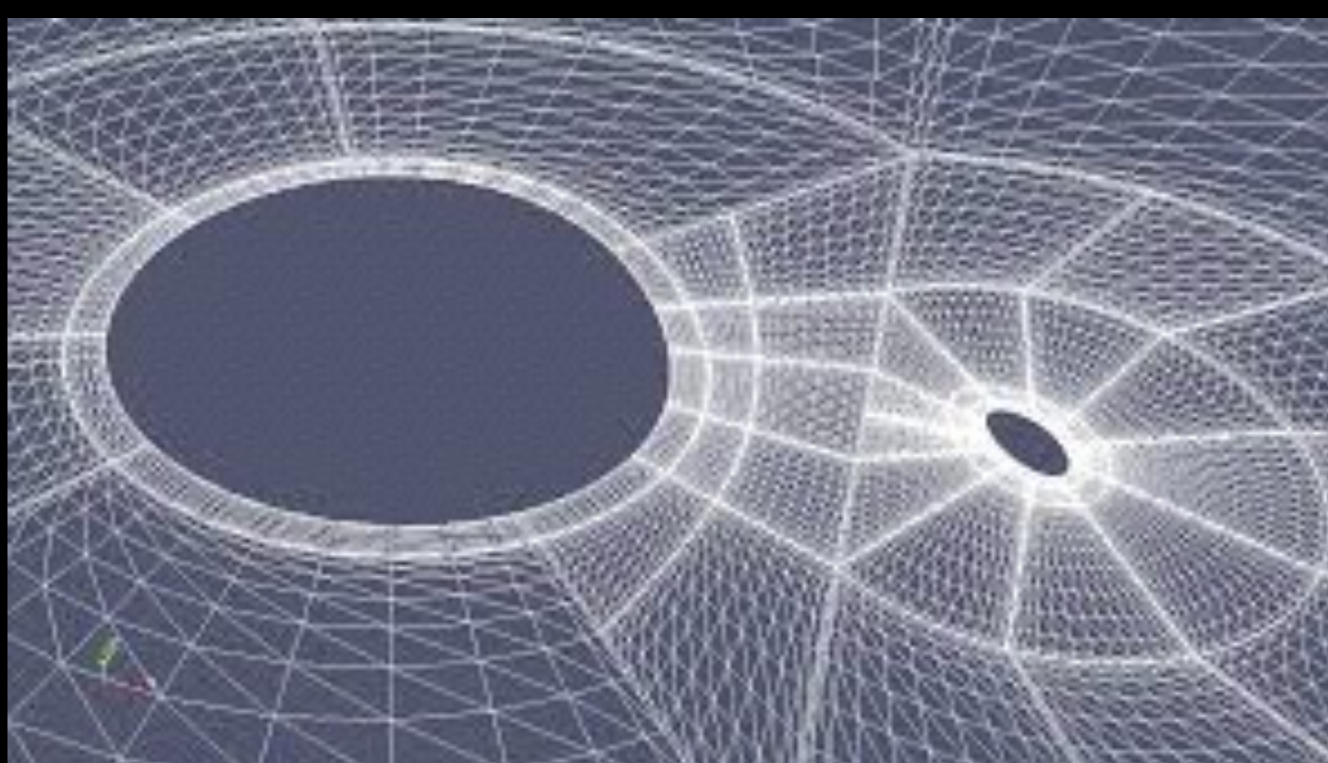


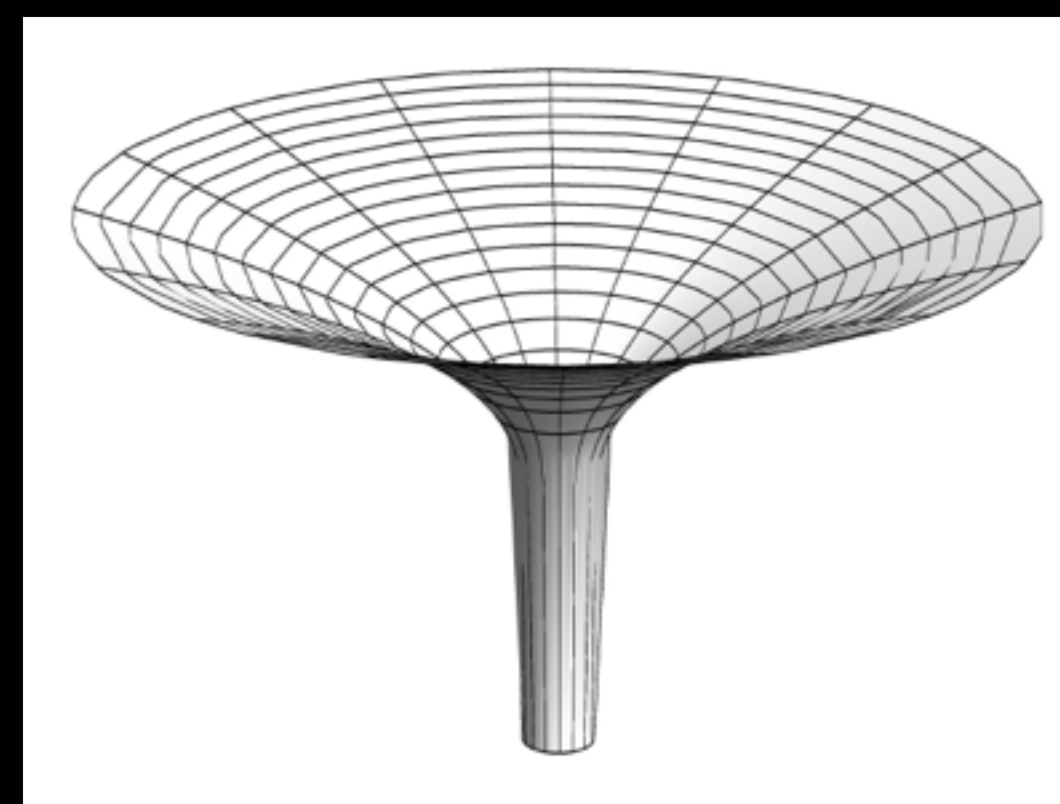


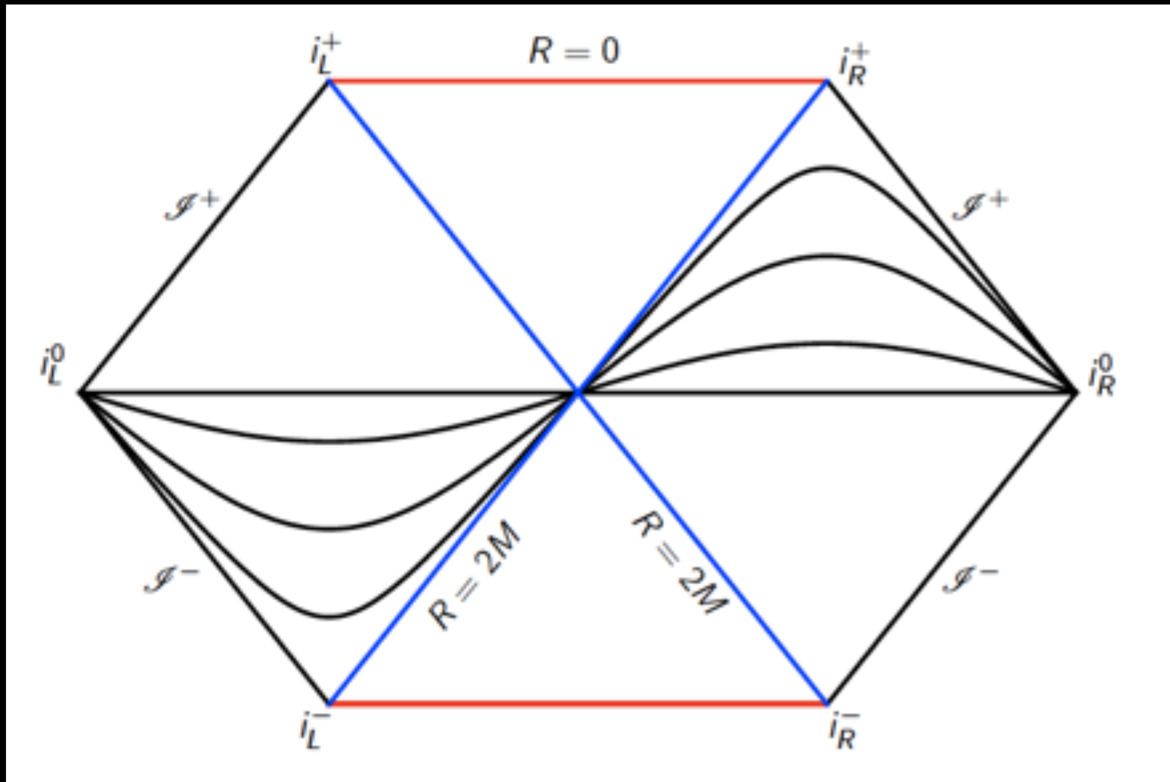
“punctures”



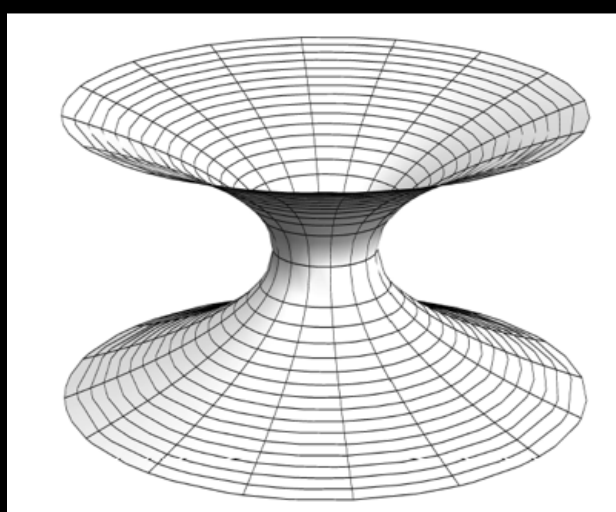
Excision



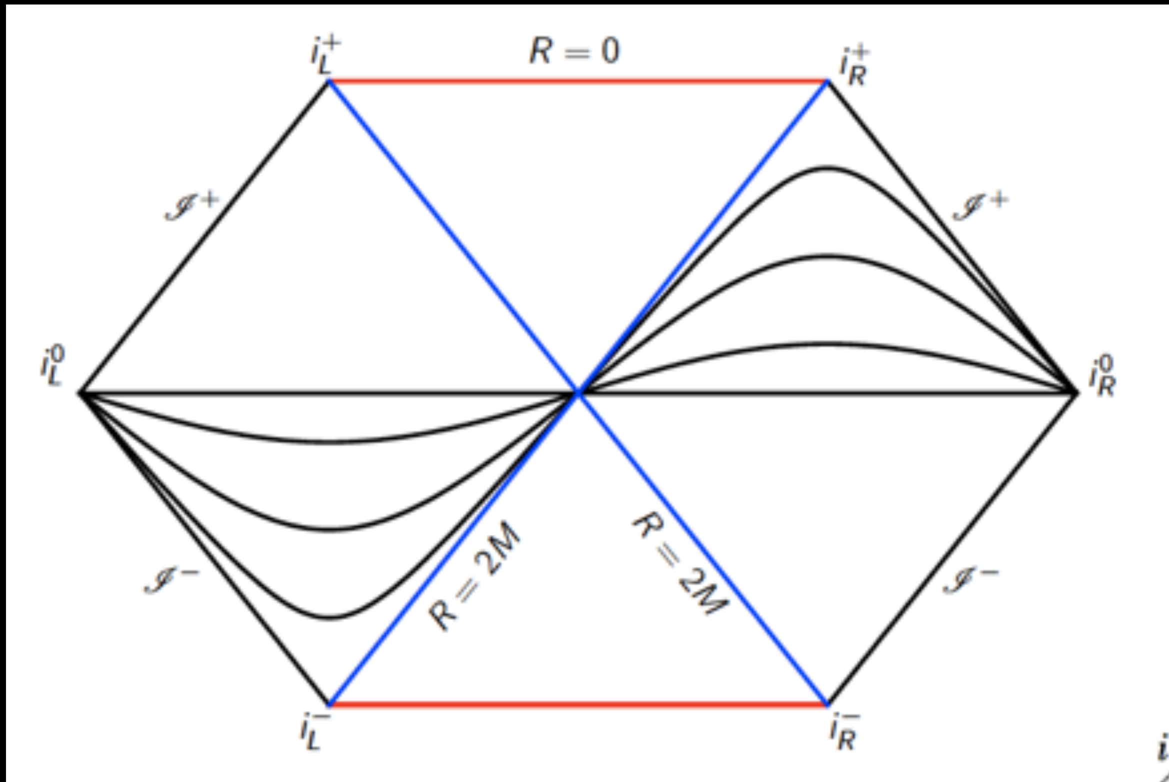


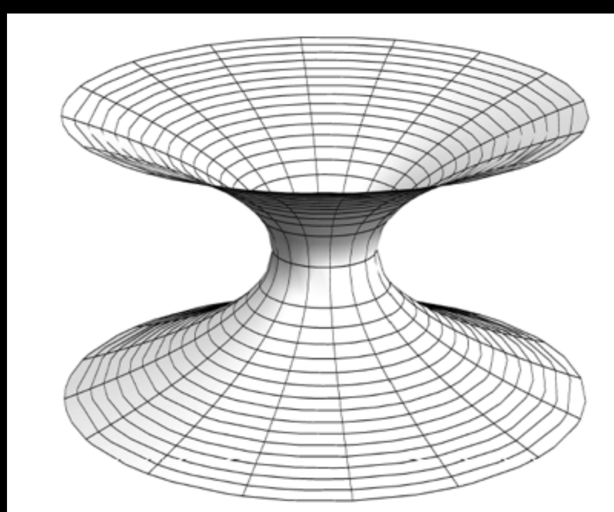


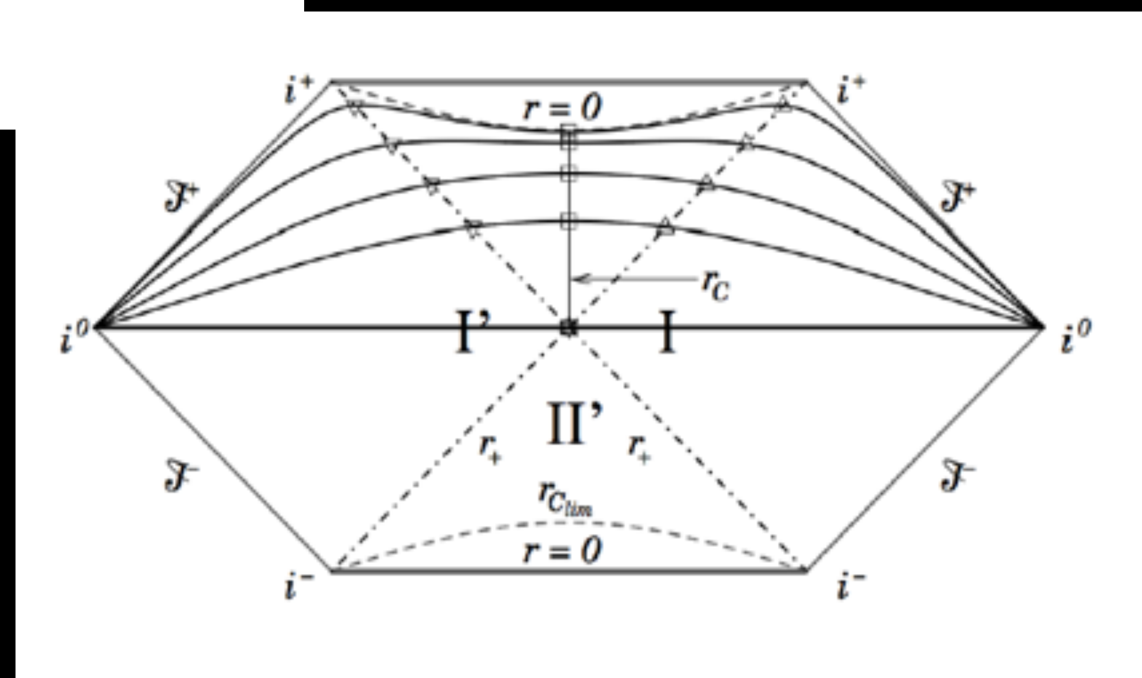
Gauge conditions



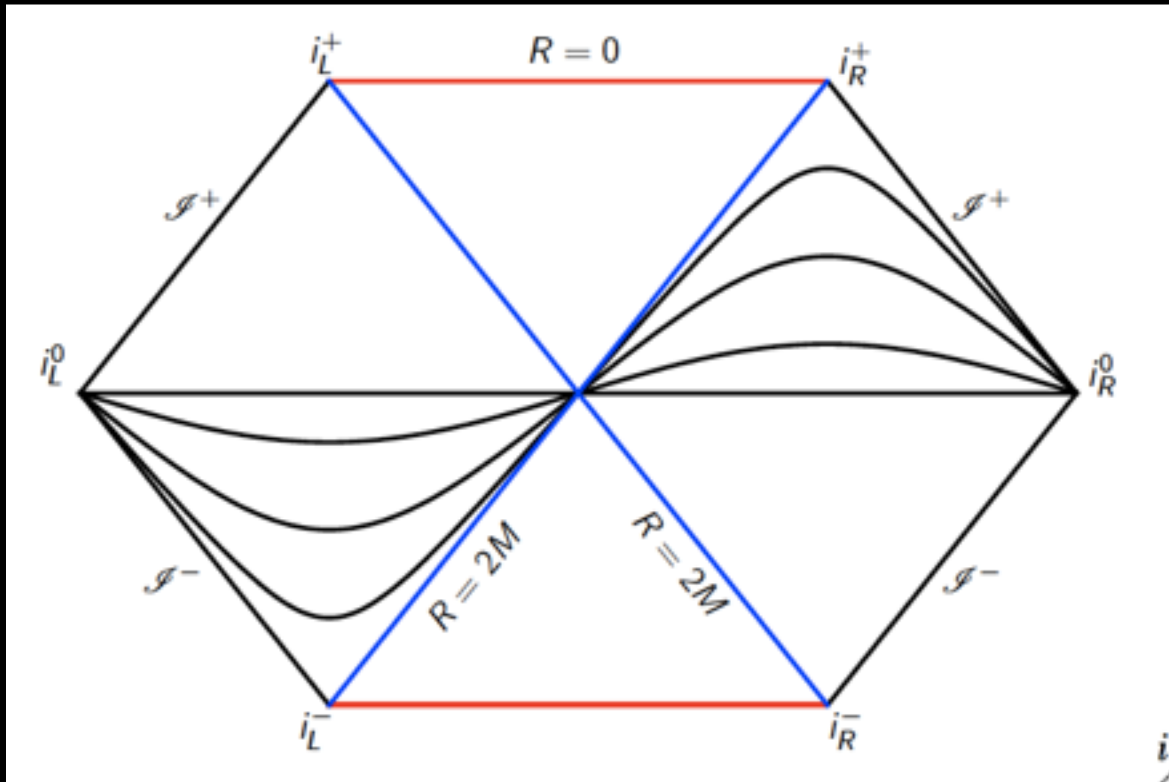
Gauge conditions

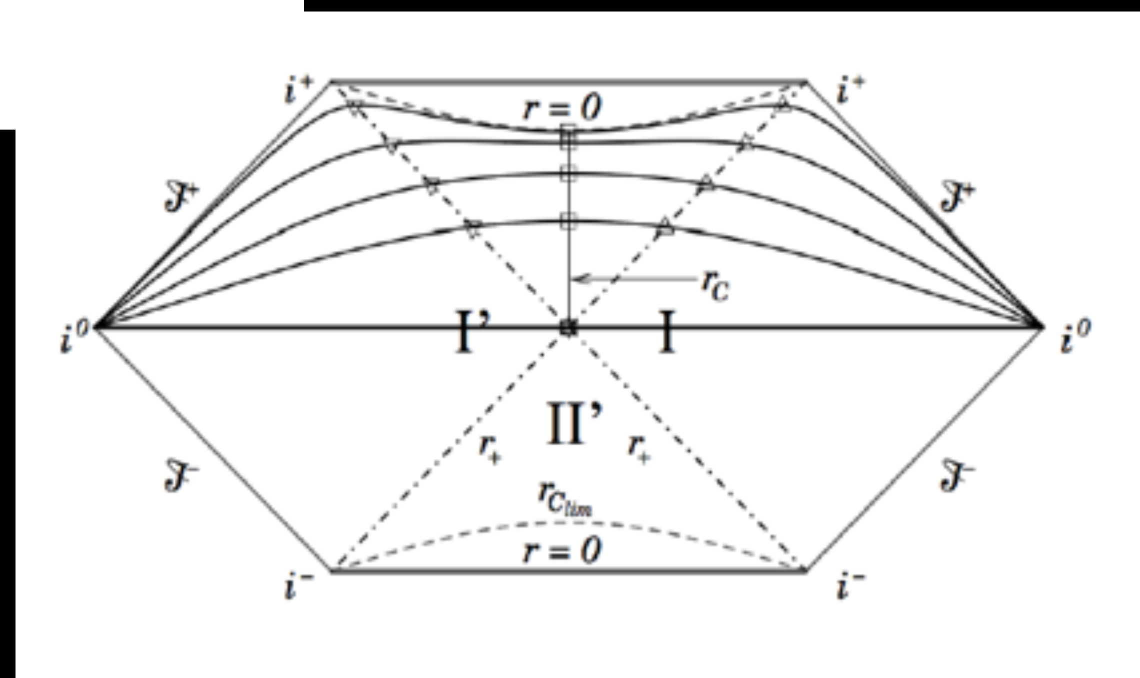


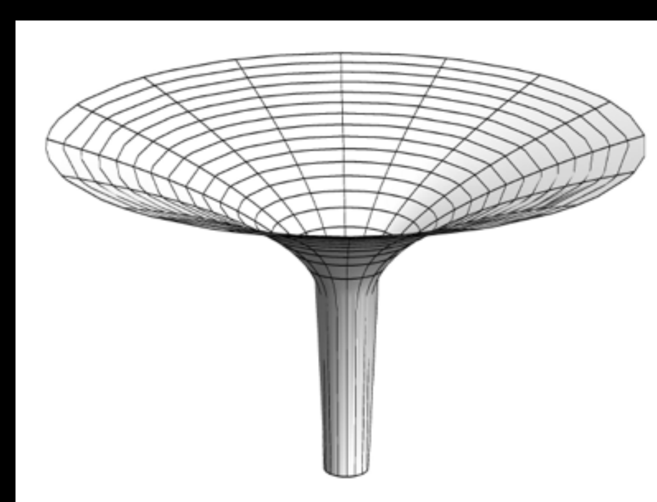


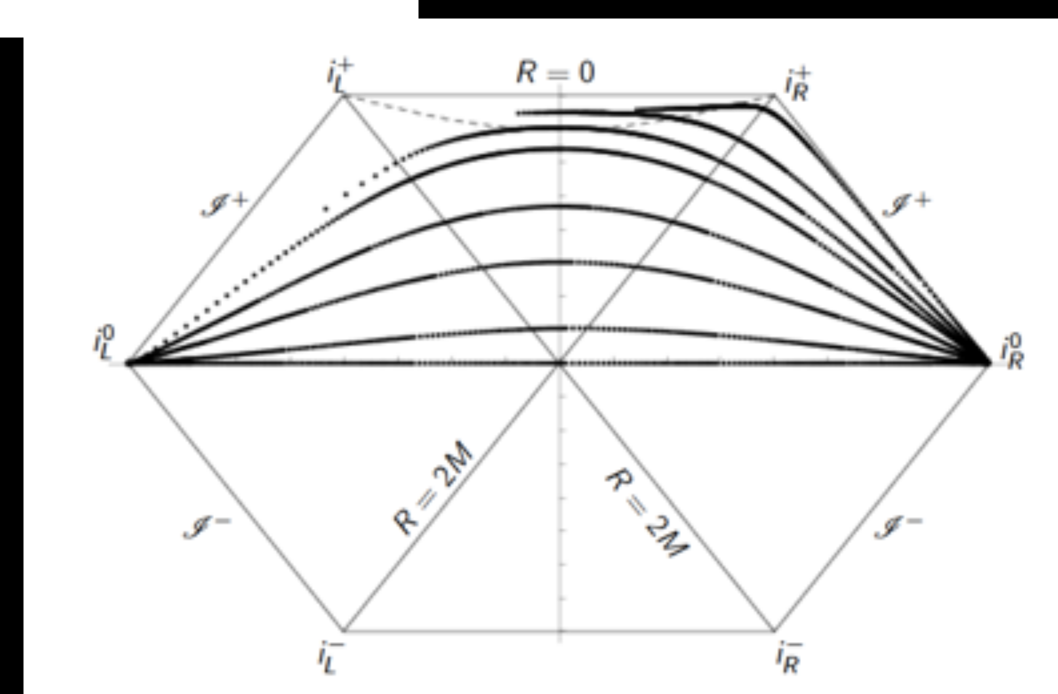


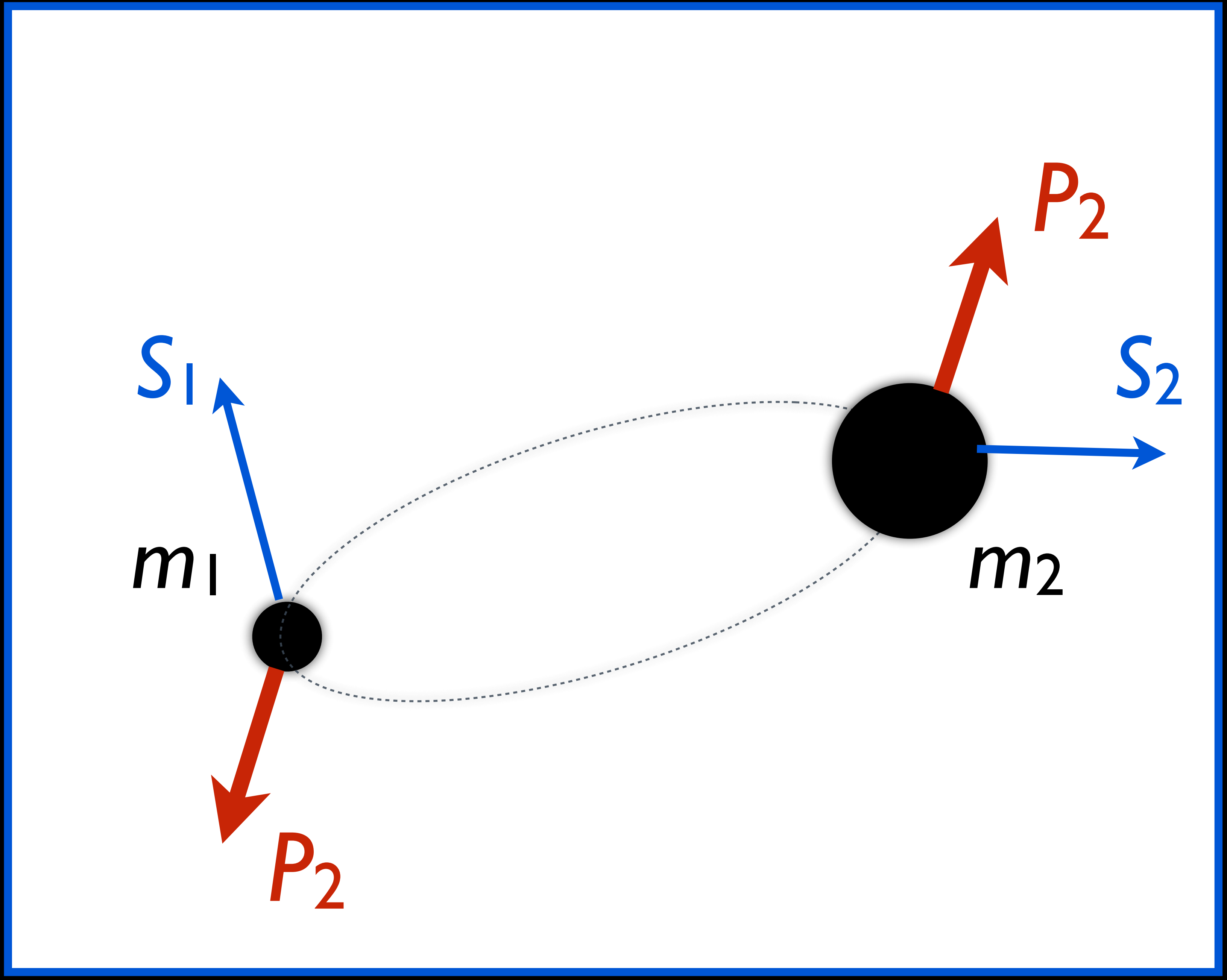
Gauge conditions





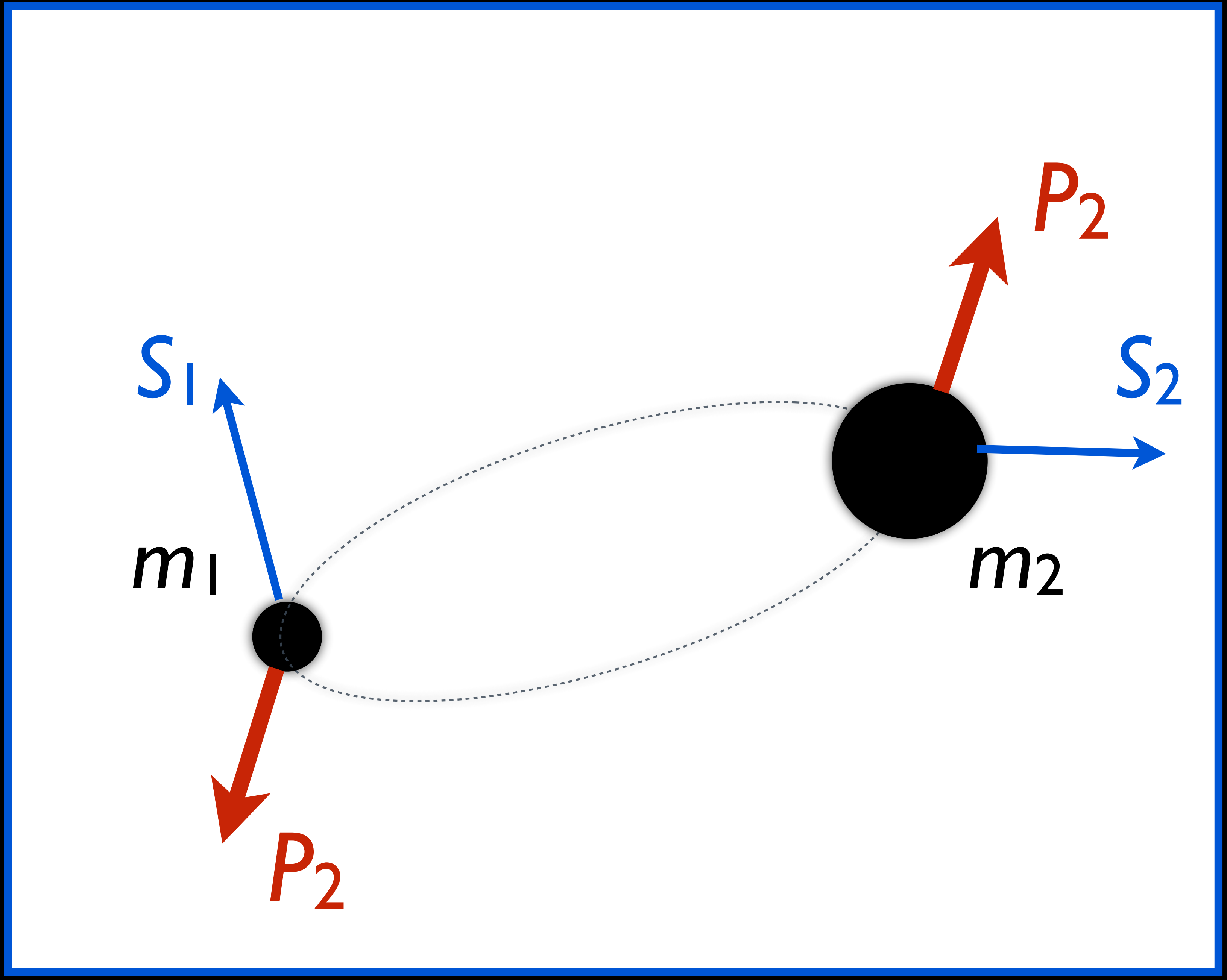


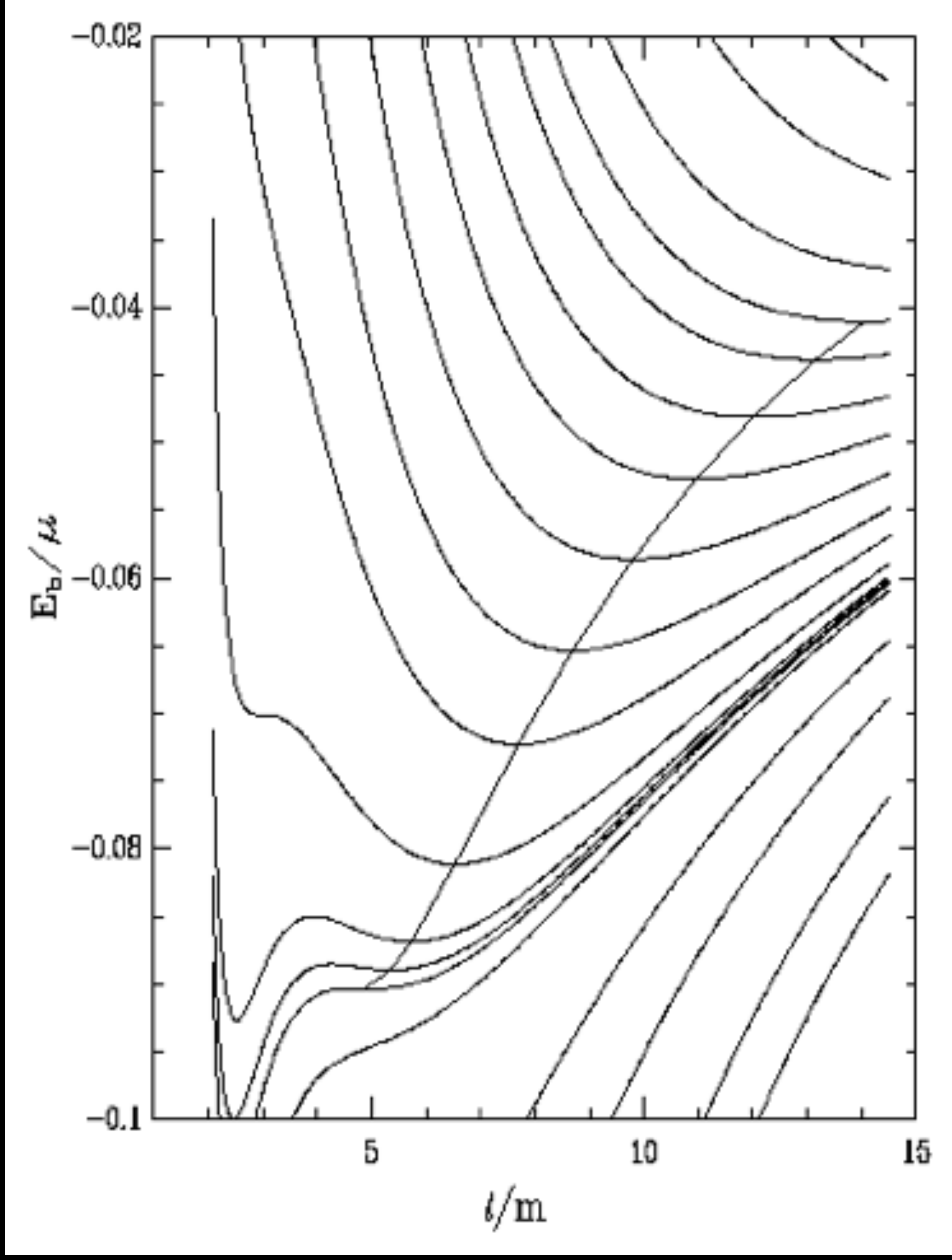




Initial conditions

Initial conditions





Boosted Three-Dimensional Black-Hole Evolutions with Singularity Excision

The accurate computational modeling of black-hole interactions is essential to the confident detection of astrophysical gravitational radiation by future space-based detectors such as LISA and by the LIGO/VIRGO/GEO complex of ground-based detectors currently under construction. The sensitivity of these detectors will be significantly enhanced if accurate computer simulations of black-hole mergers can produce predictions of radiation waveforms [1]. The Binary Black Hole Grand Challenge Alliance [2] was funded in September 1993 to develop the computational infrastructure for accurate simulations of the coalescence of black-hole binaries. The primary objective of the resulting code will be the prediction of waveforms from binary black-hole mergers. In this Letter we report on an important step towards achieving such simulations.

Boosted Three-Dimensional Black-Hole Evolutions with Singularity Excision

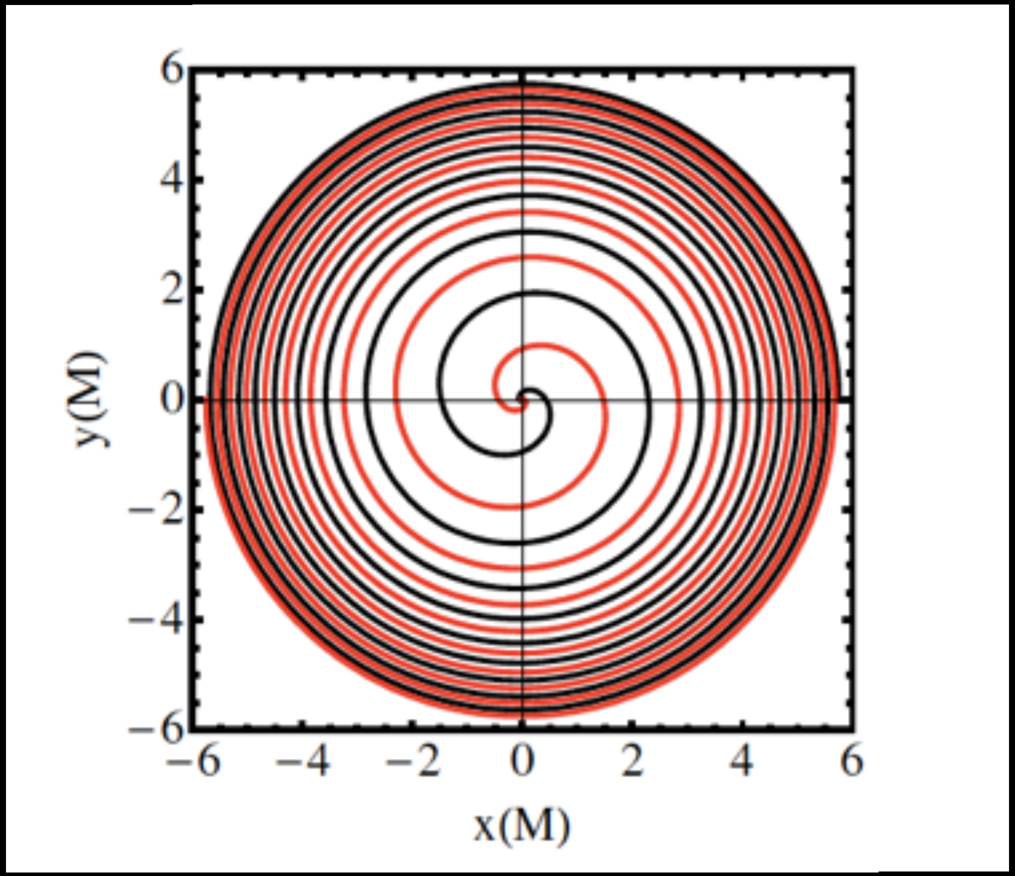
[redacted]

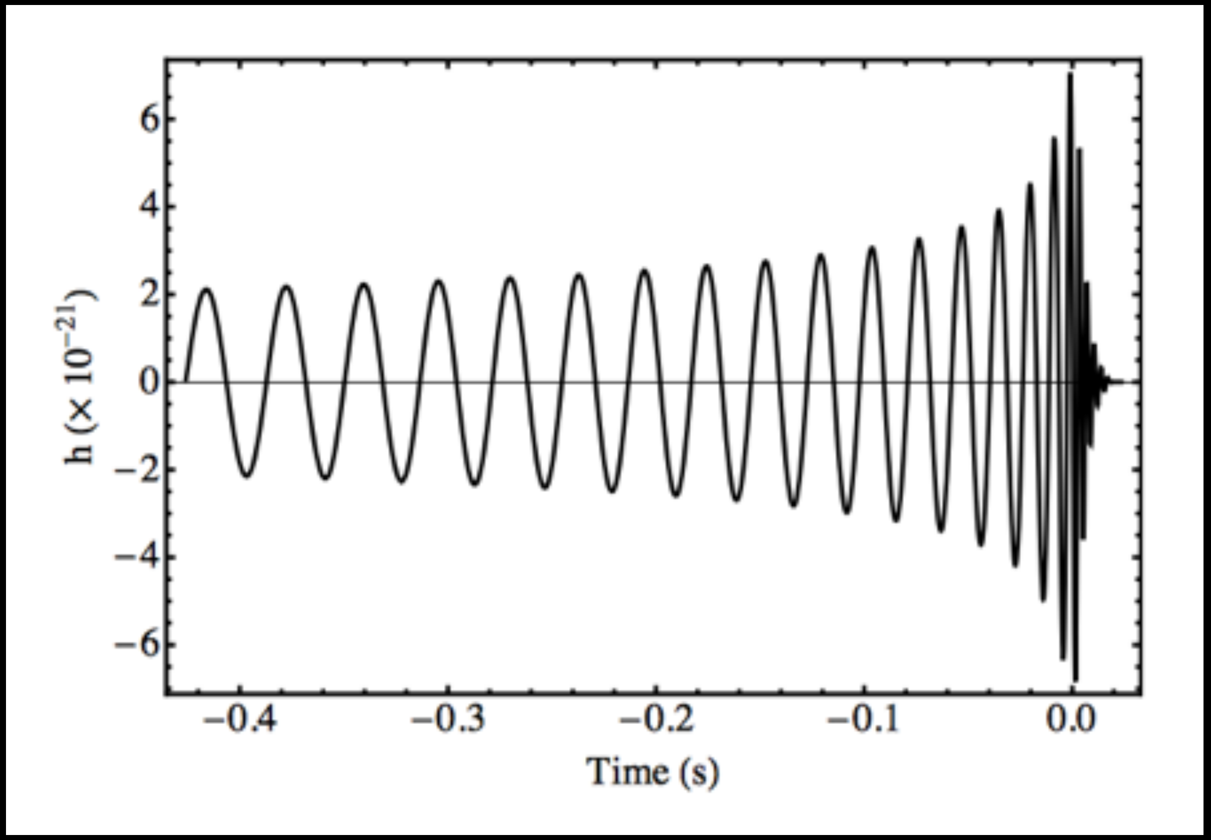
Binary black-hole interactions provide potentially the strongest source of gravitational radiation for detectors currently under development. We present some results from the Binary Black Hole Grand Challenge Alliance three-dimensional Cauchy evolution module. These constitute essential steps towards modeling such interactions and predicting gravitational radiation waveforms. We report on single black-hole evolutions and the first successful demonstration of a black hole moving freely through a three-dimensional computational grid via a Cauchy evolution: a hole moving near $6M$ at $0.1c$ during a total evolution of duration near $60M$. [S0031-9007(98)05652-X]

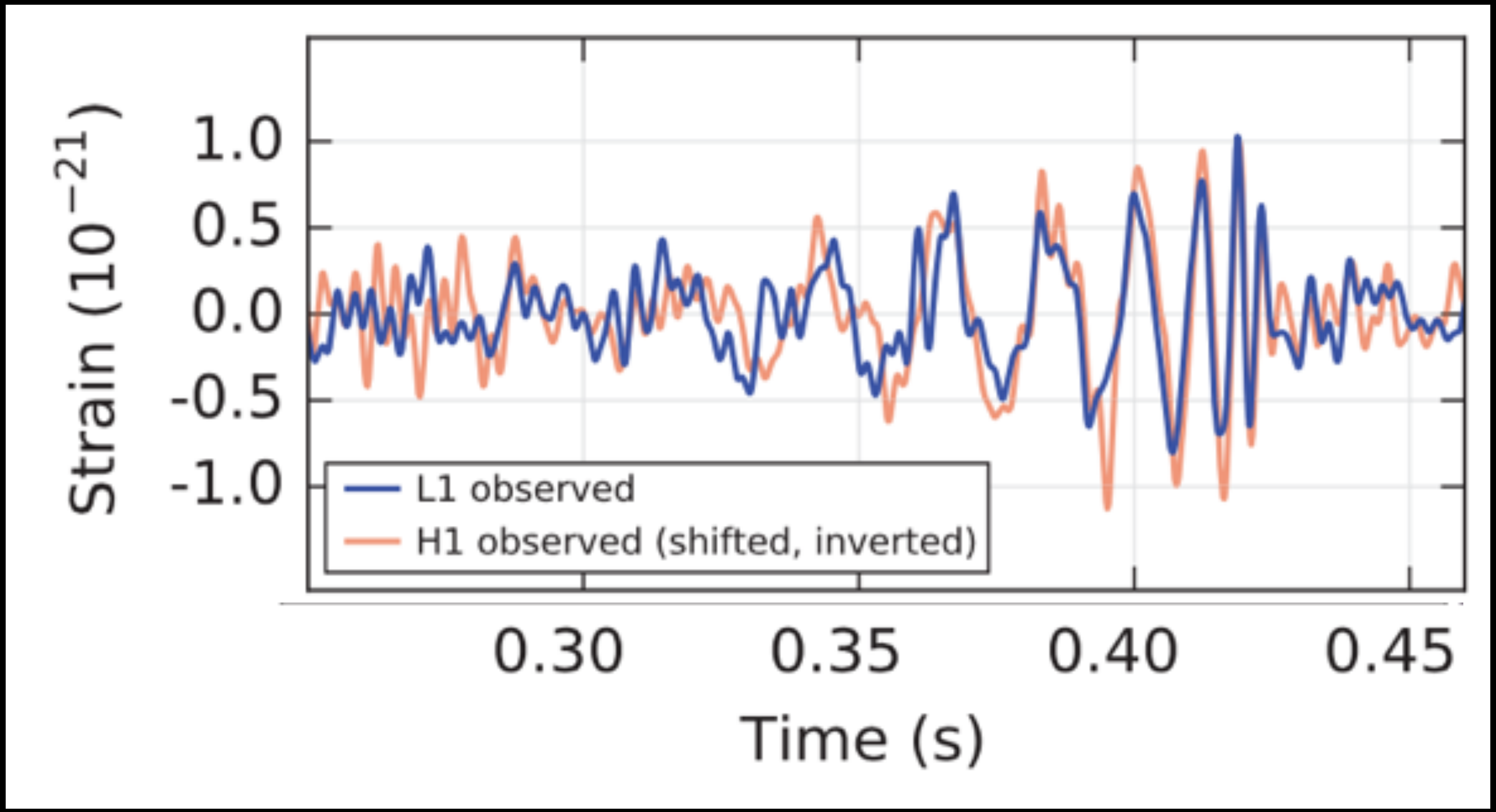
2005: Breakthrough!

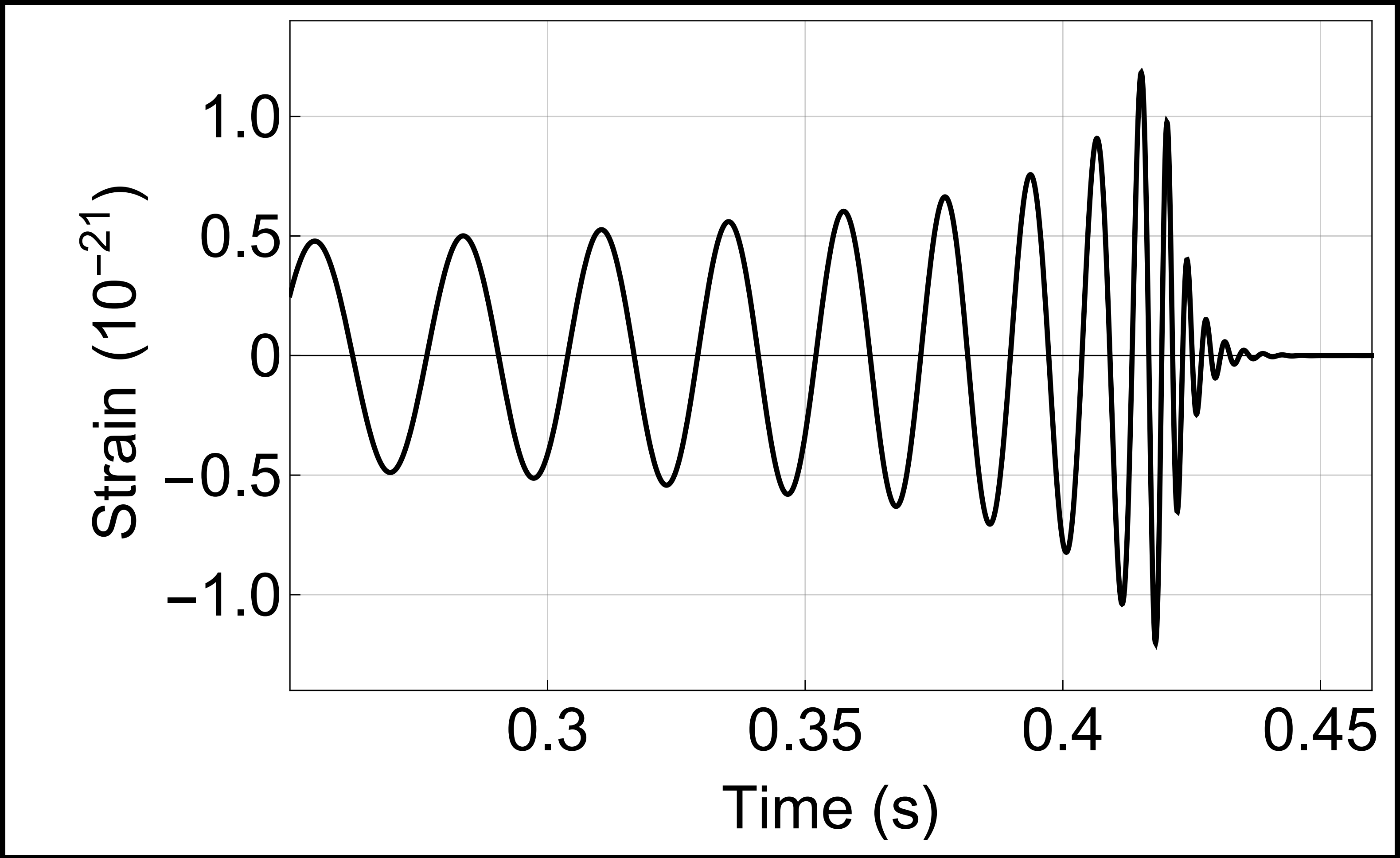
Pretorius (July): Generalised harmonic formalism

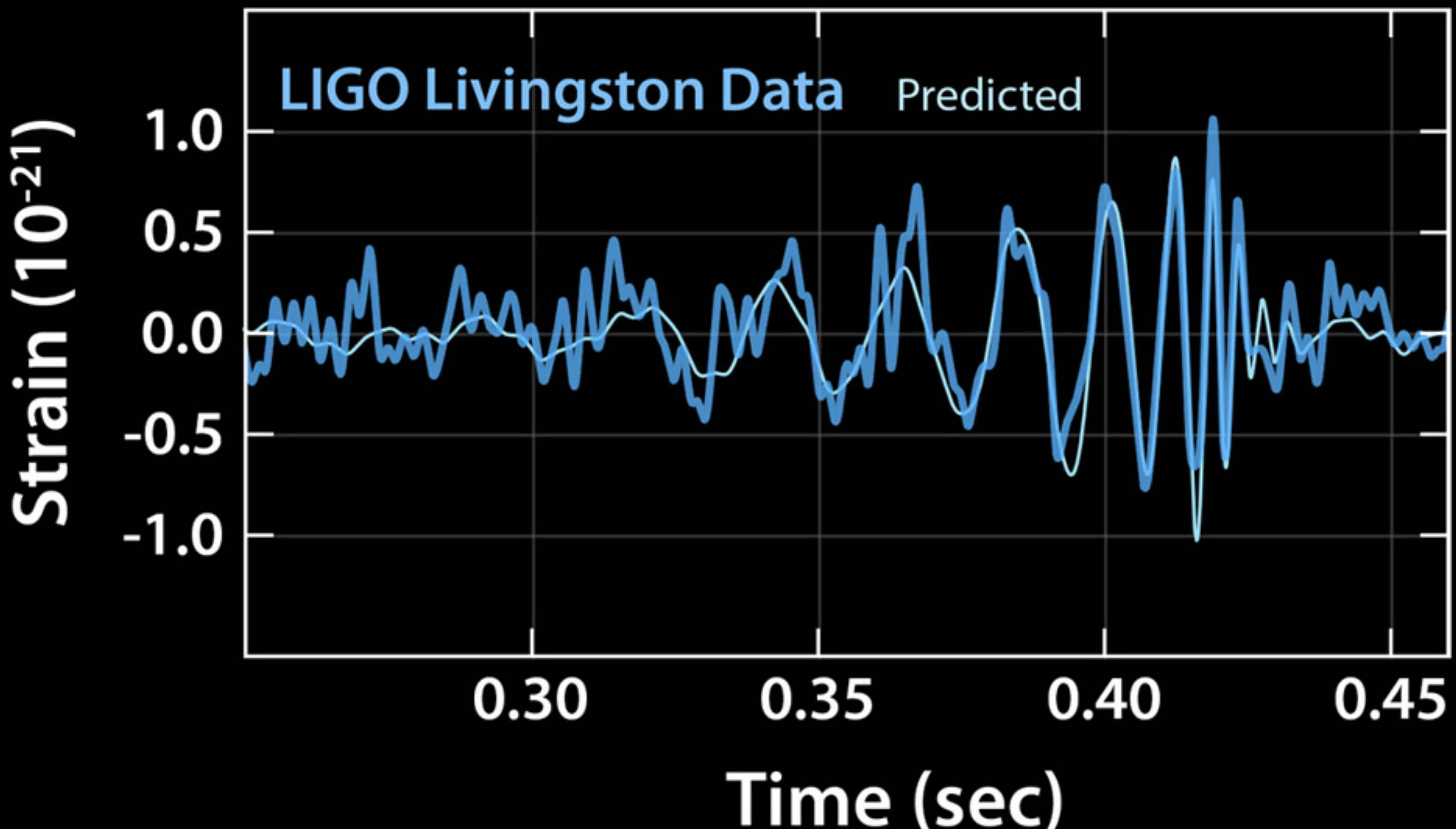
NASA-Goddard and Brownsville-Texas (November): moving-puncture method.






Strain (10^{-21})
1.0
0.5
0.0
-0.5
-1.0
L1 observed
H1 observed (shifted, inverted)
0.30
0.35
0.40
0.45
Time (s)


Strain (10^{-21})
1.0
0.5
0
−0.5
−1.0
0.3
0.35
0.4
0.45
Time (s)


LIGO Livingston Data
Predicted
Strain (10^-21)
1.0
0.5
0.0
-0.5
-1.0
0.30
0.35
0.40
0.45
Time (sec)

Waveform modelling
and
black-hole
measurements

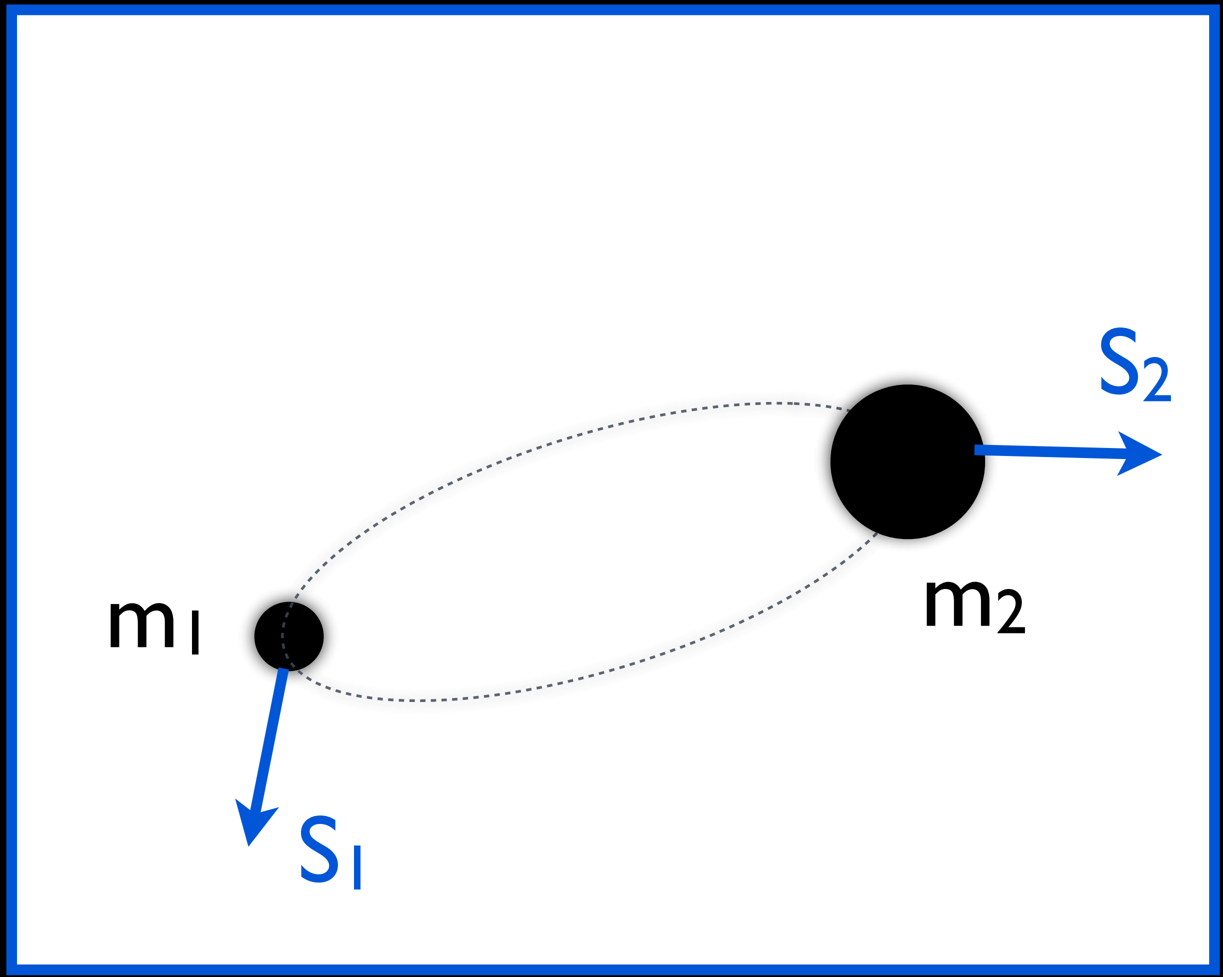
Masses: m_1, m_2

Spins: S_1, S_2

(8 parameters)

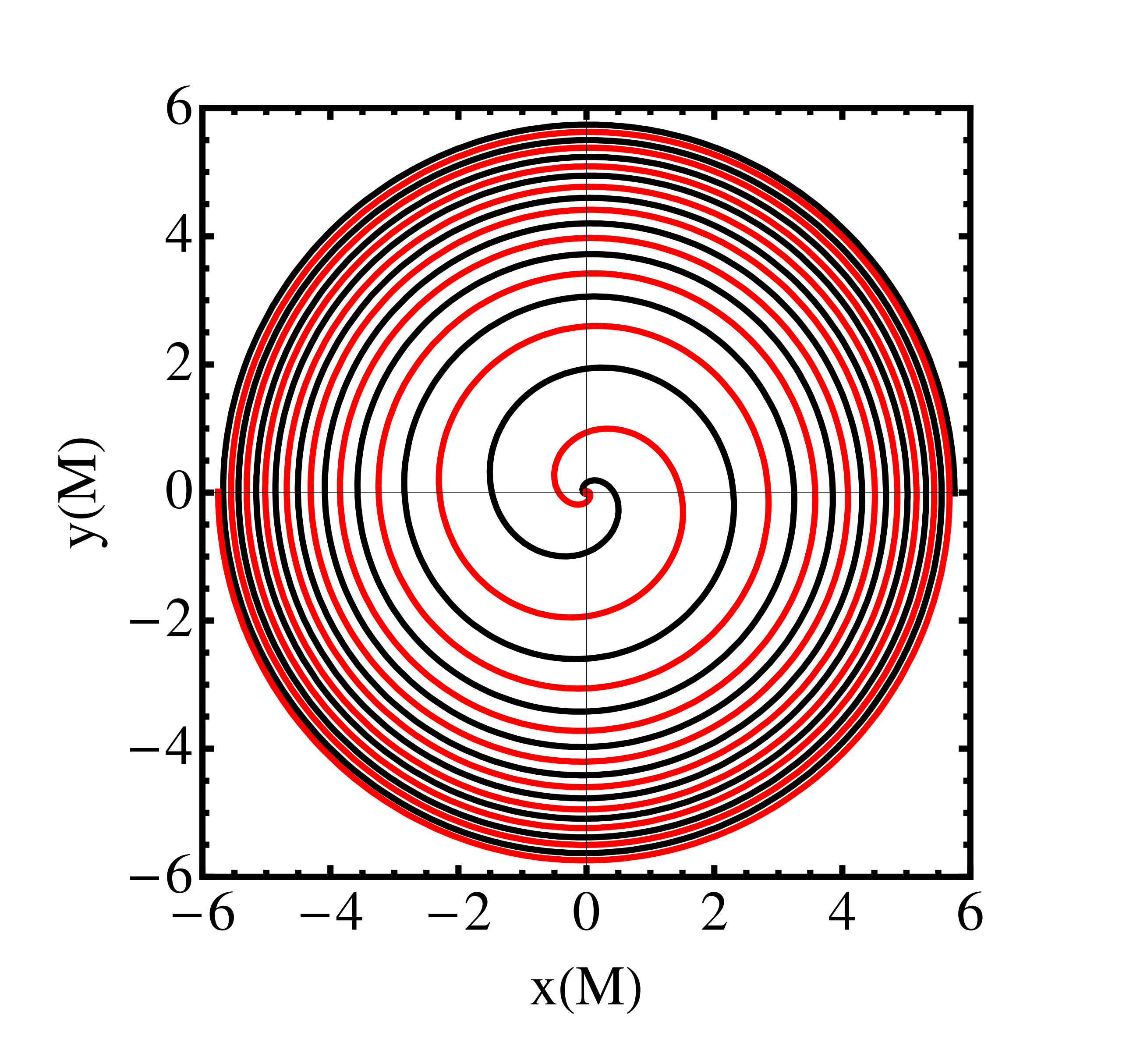
useful combinations:

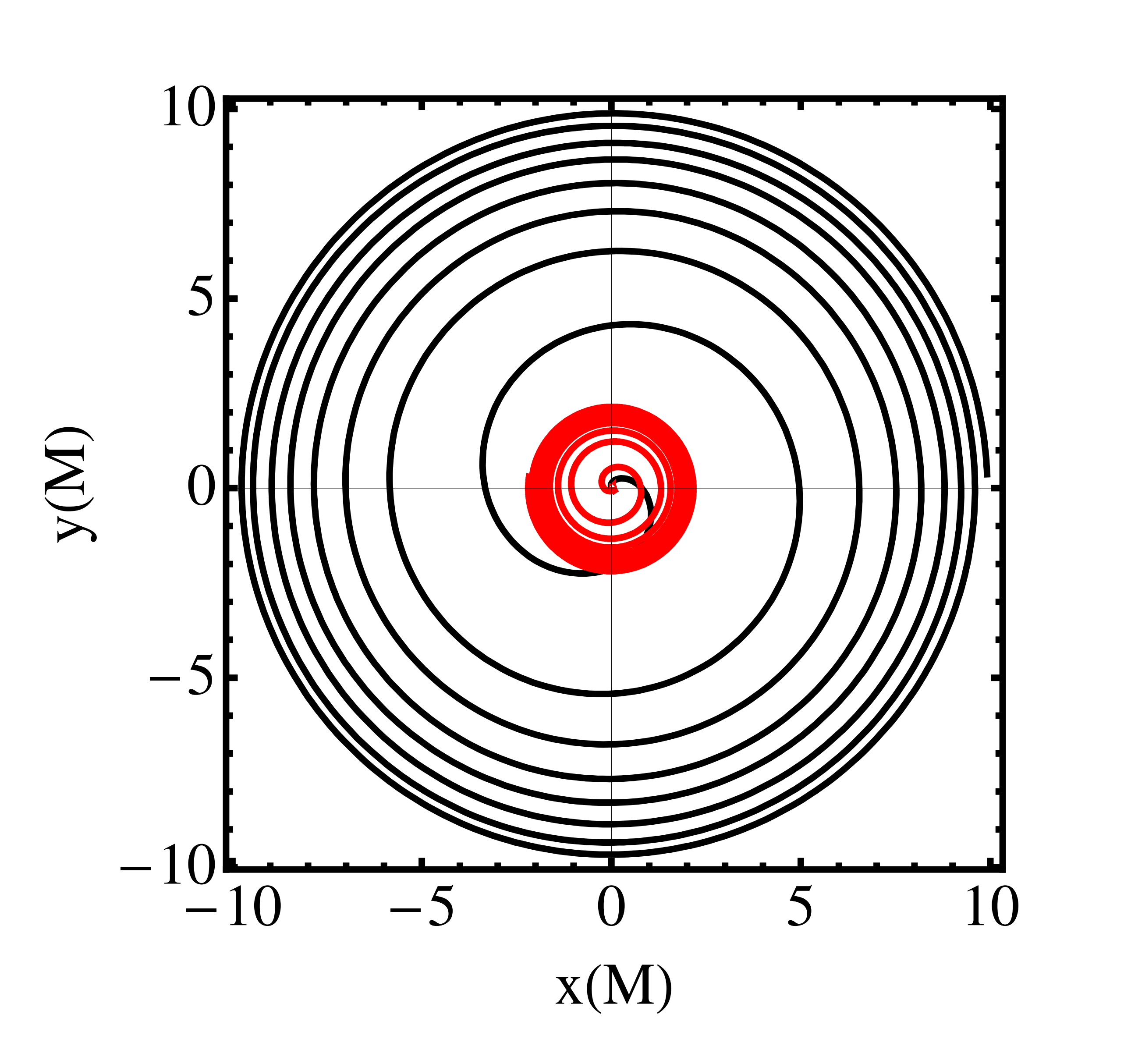
$$M = m_1 + m_2$$
$$q = m_2 / m_1$$
$$\eta = m_1 m_2 / M^2$$
$$\chi = S/m^2$$



Plus: distance, sky location, orientation, polarisation

Nonspinning black holes





Amplitude:

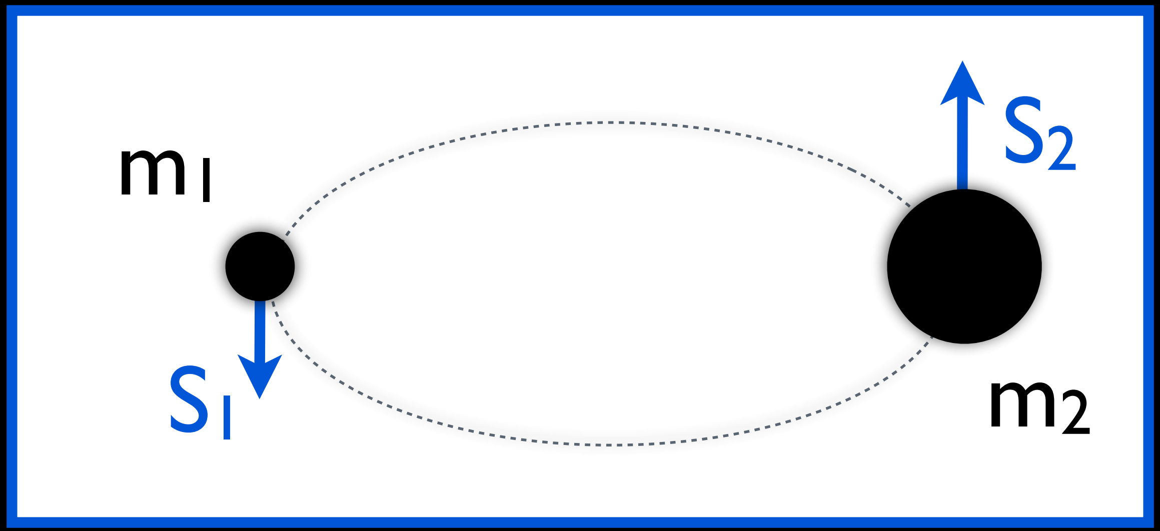
Optimally oriented (face on)

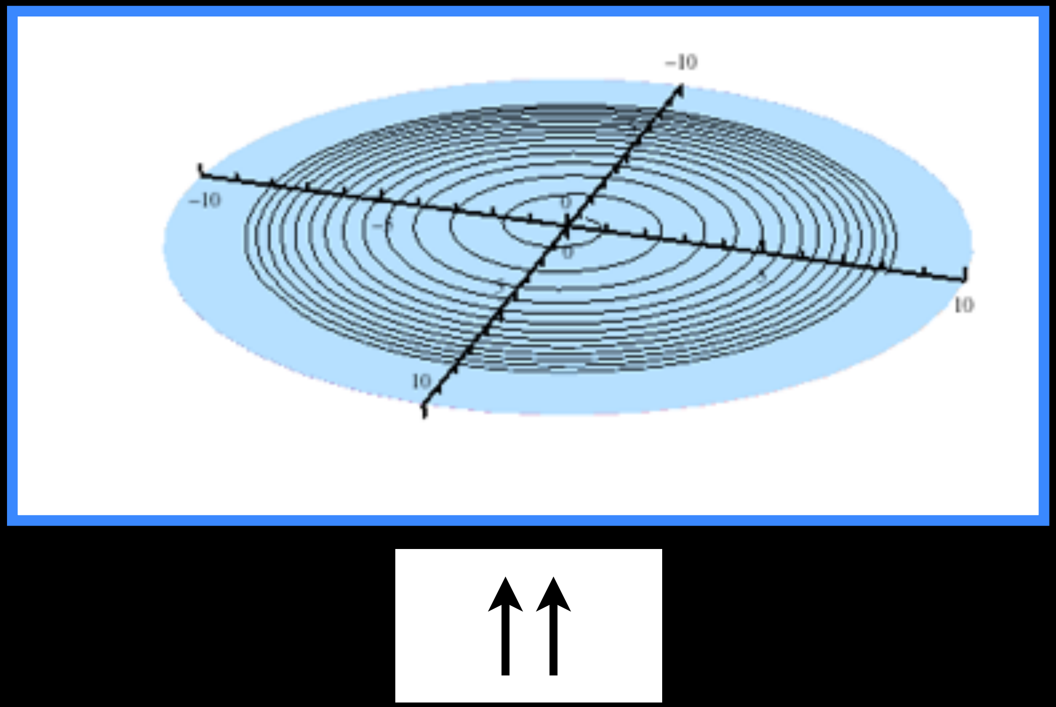
Edge on

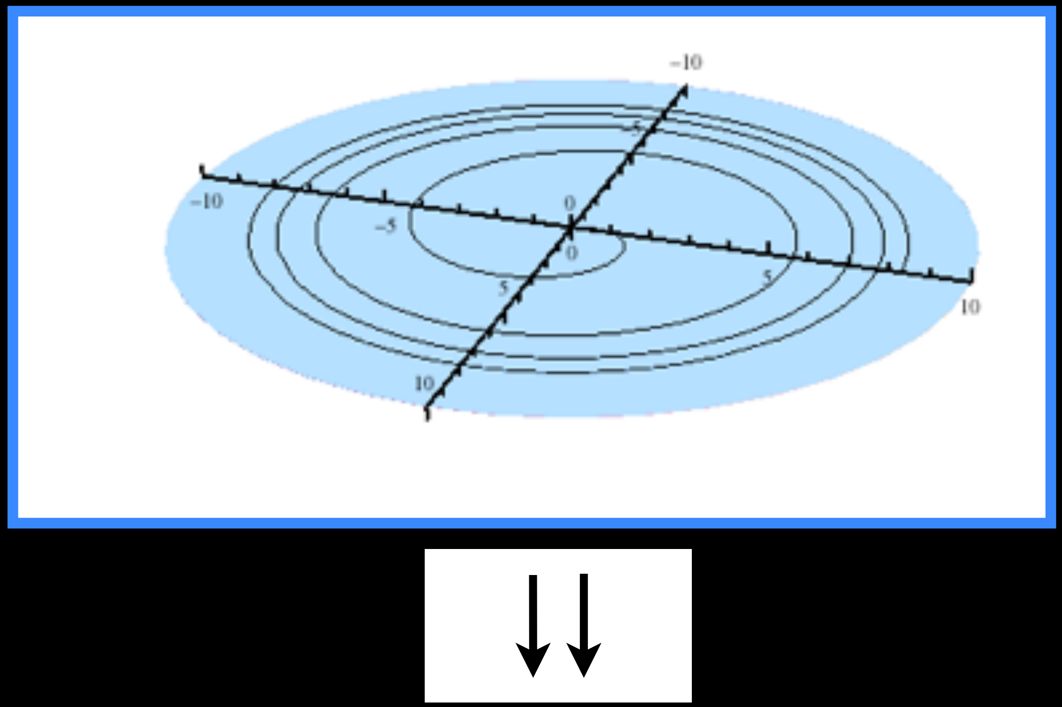
Signal shape is independent of orientation

Key information is in the phasing

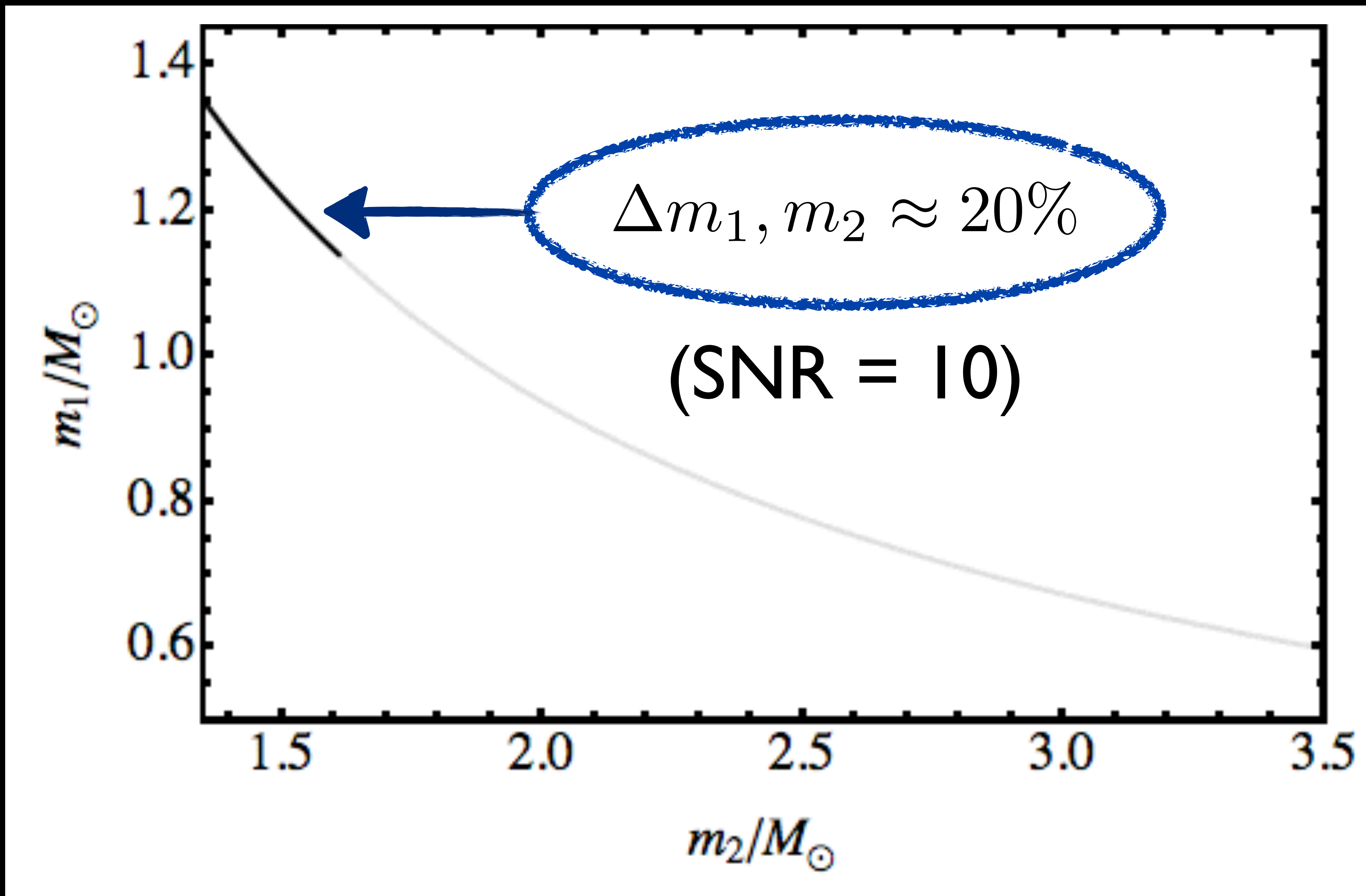
Aligned spins





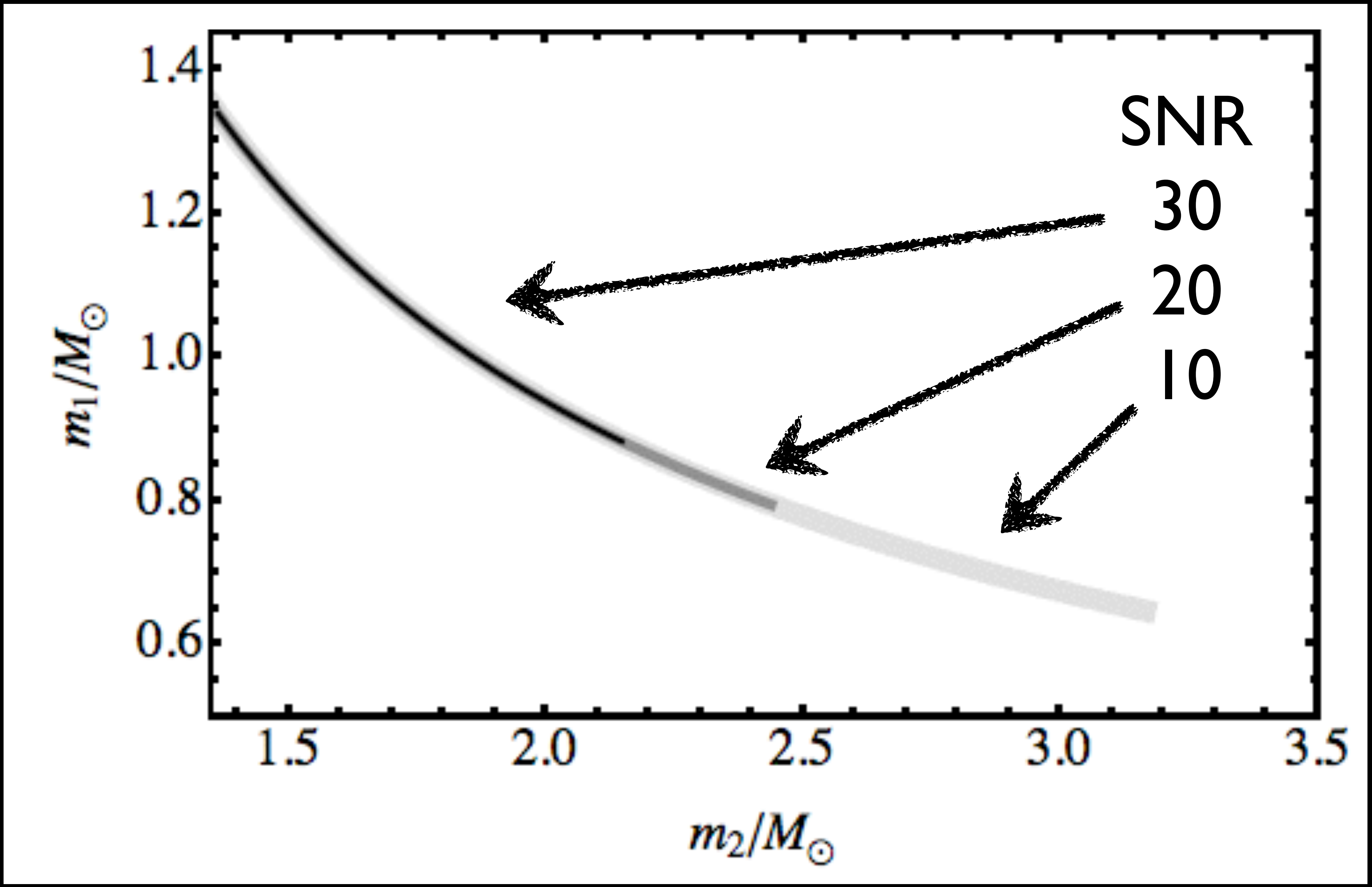


Mass measurements (non-spinning)

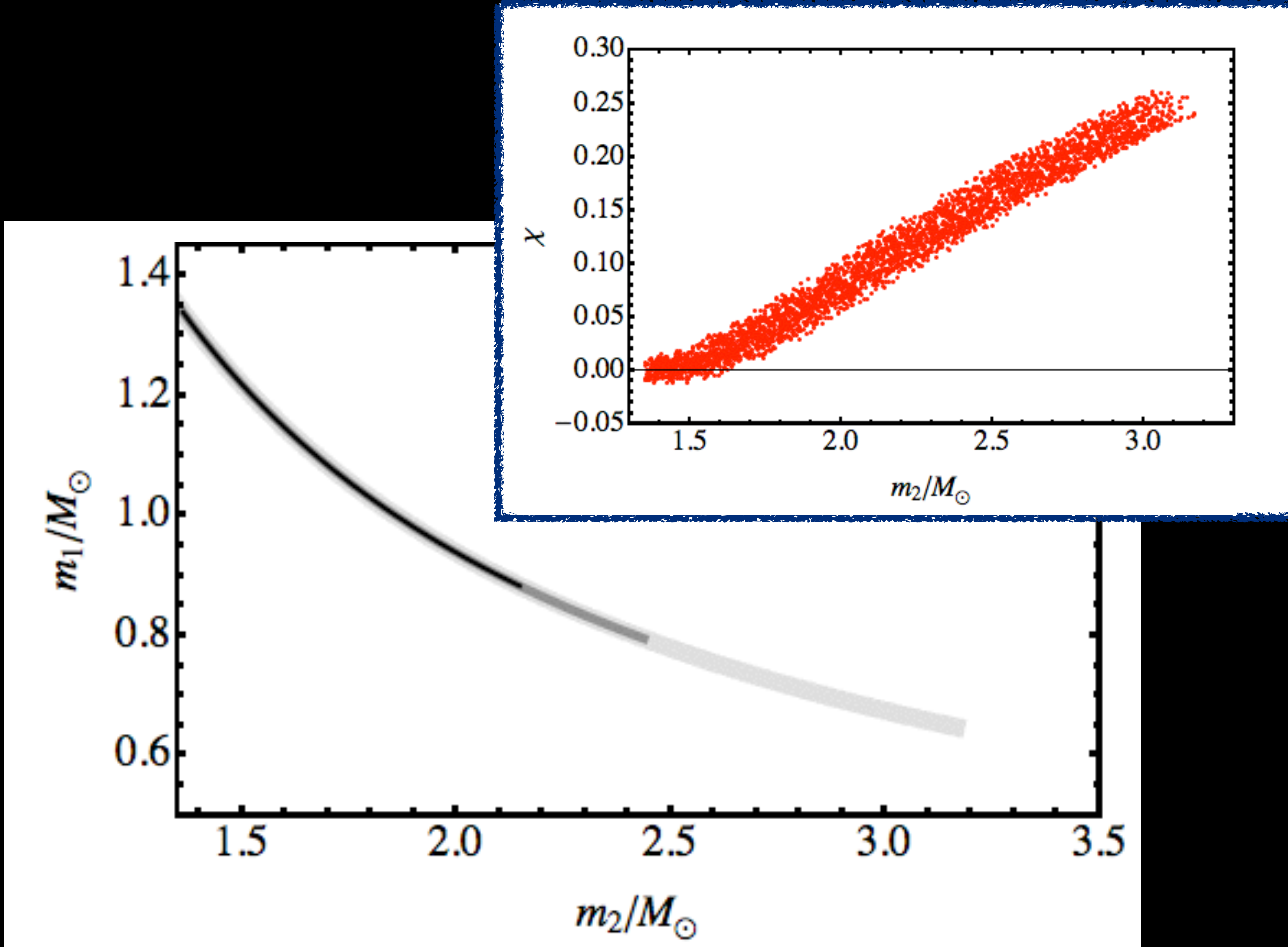


[Hannam, *et. al* (2013)]

Aligned spins



[Hannam, *et. al* (2013)]



[Hannam, *et. al* (2013)]

What is “χ”?

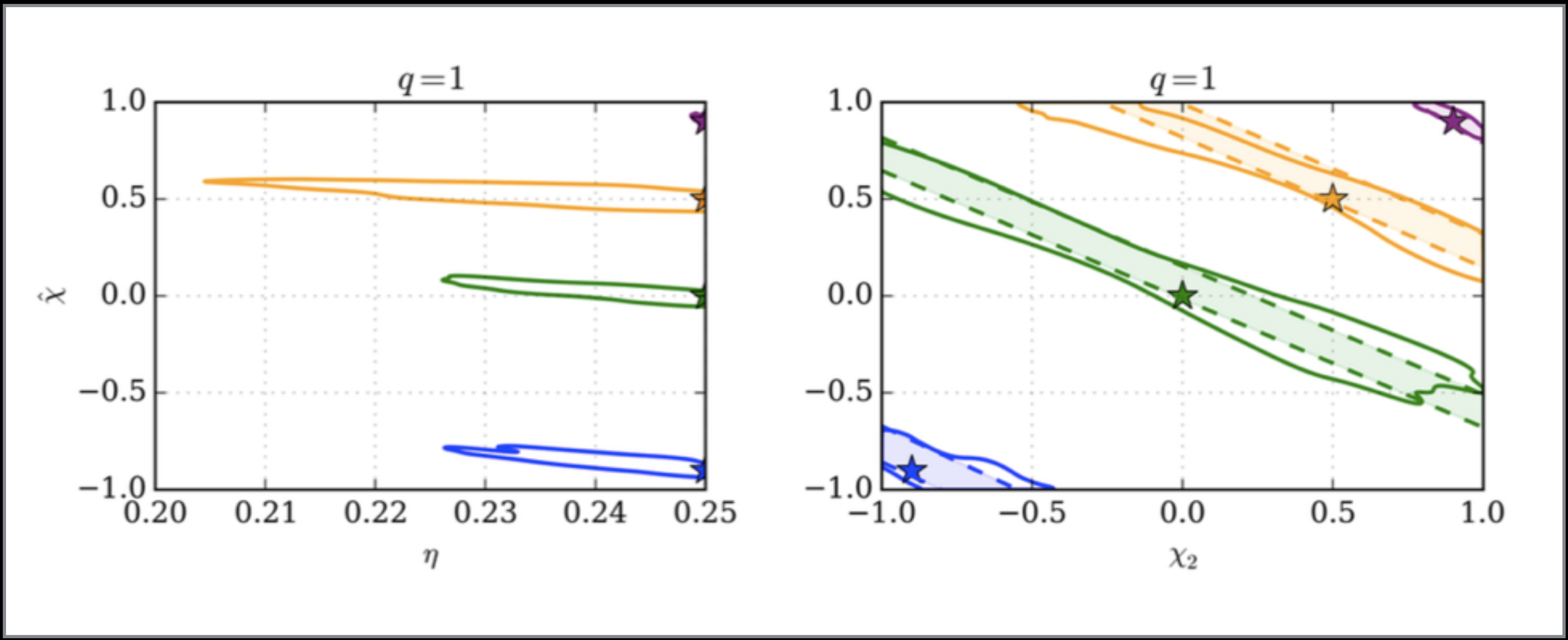
χ is a weighted sum of the two spins

χ is the dominant spin effect on the phasing

The individual spins have only a weak effect

$$\chi = \frac{m_1\chi_1 + m_2\chi_2}{M} - \frac{76\eta}{226}(\chi_1 + \chi_2)$$

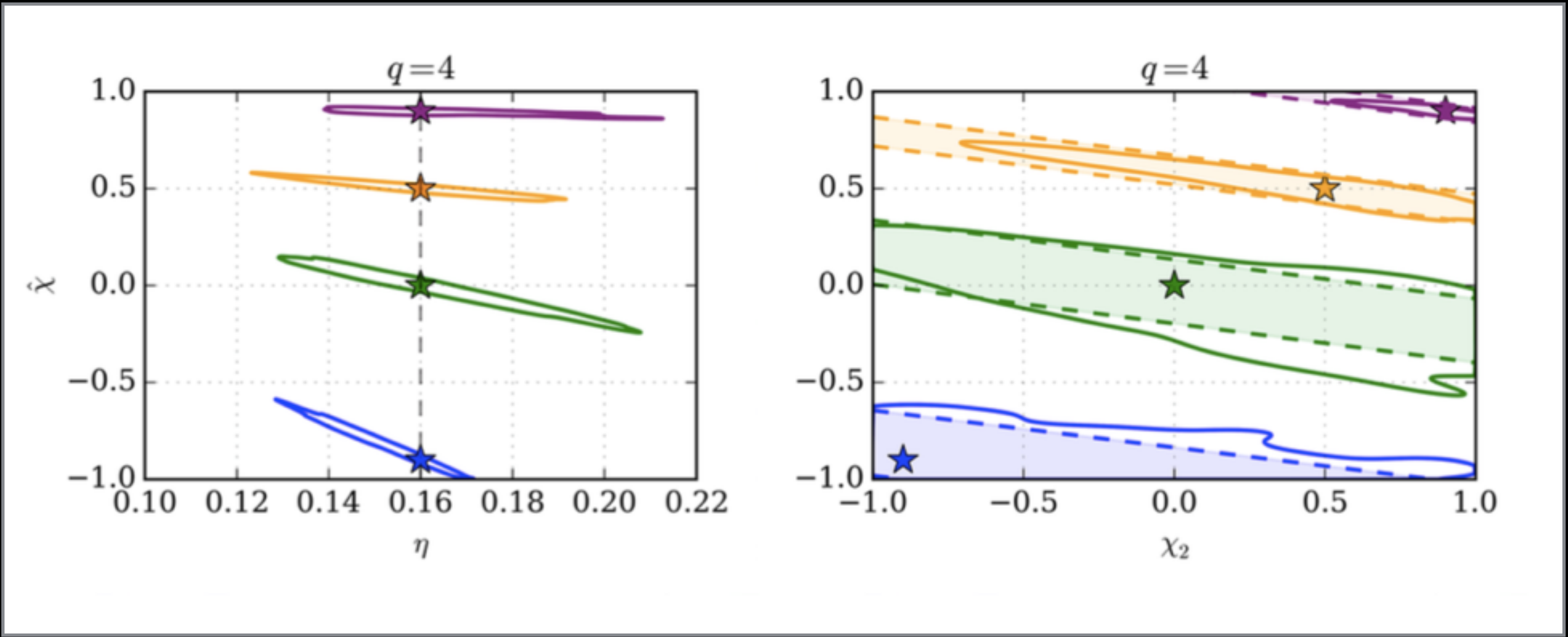
(50-solar-mass, equal spins)



[Puerrer, Hannam, Ohme (2016)]

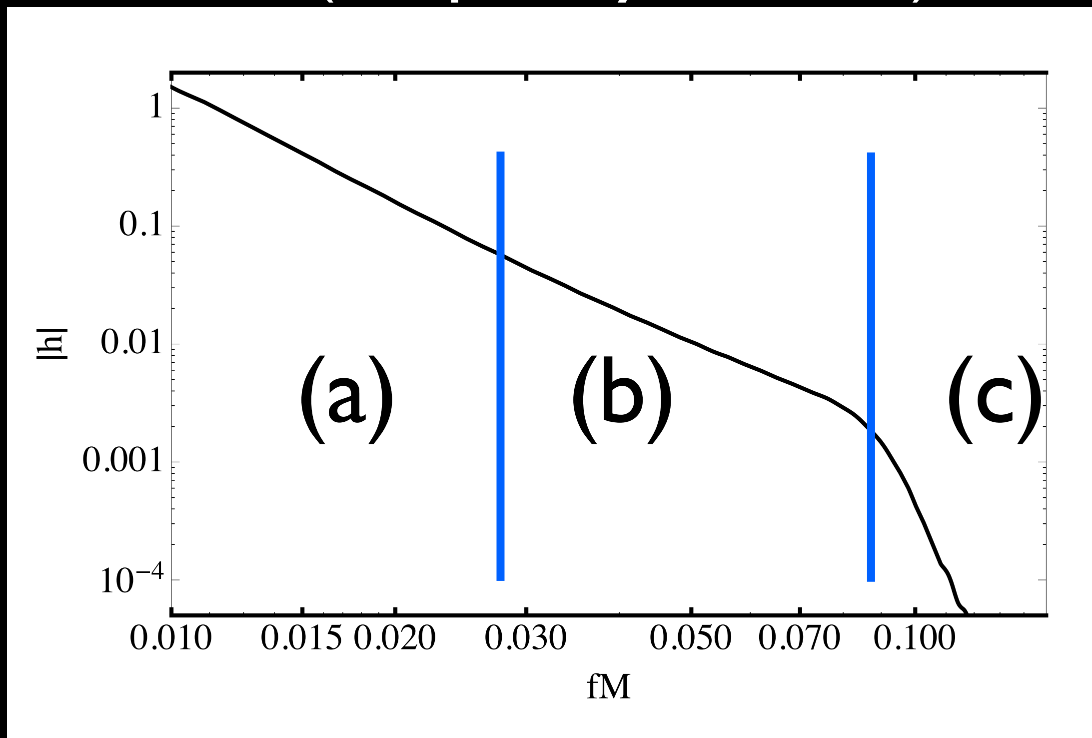
$$\chi = \frac{m_1\chi_1 + m_2\chi_2}{M} - \frac{76\eta}{226}(\chi_1 + \chi_2)$$

(50-solar-mass, equal spins)



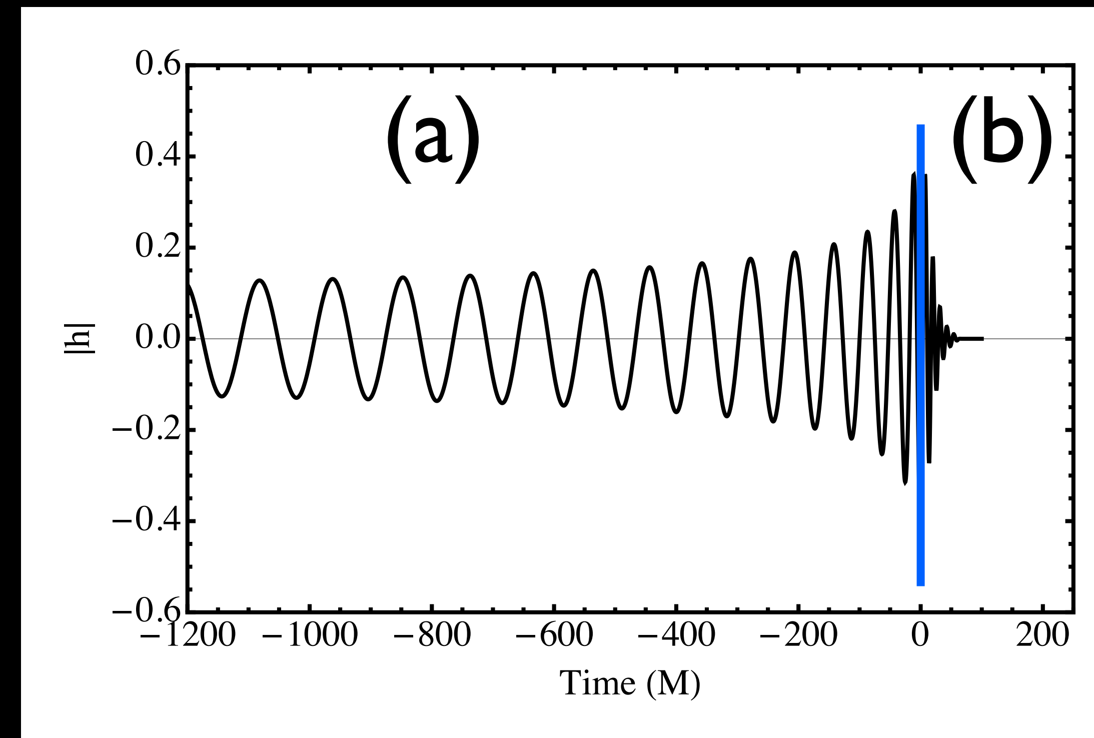
[Puerrer, Hannam, Ohme (2016)]

Phenom
(frequency domain)

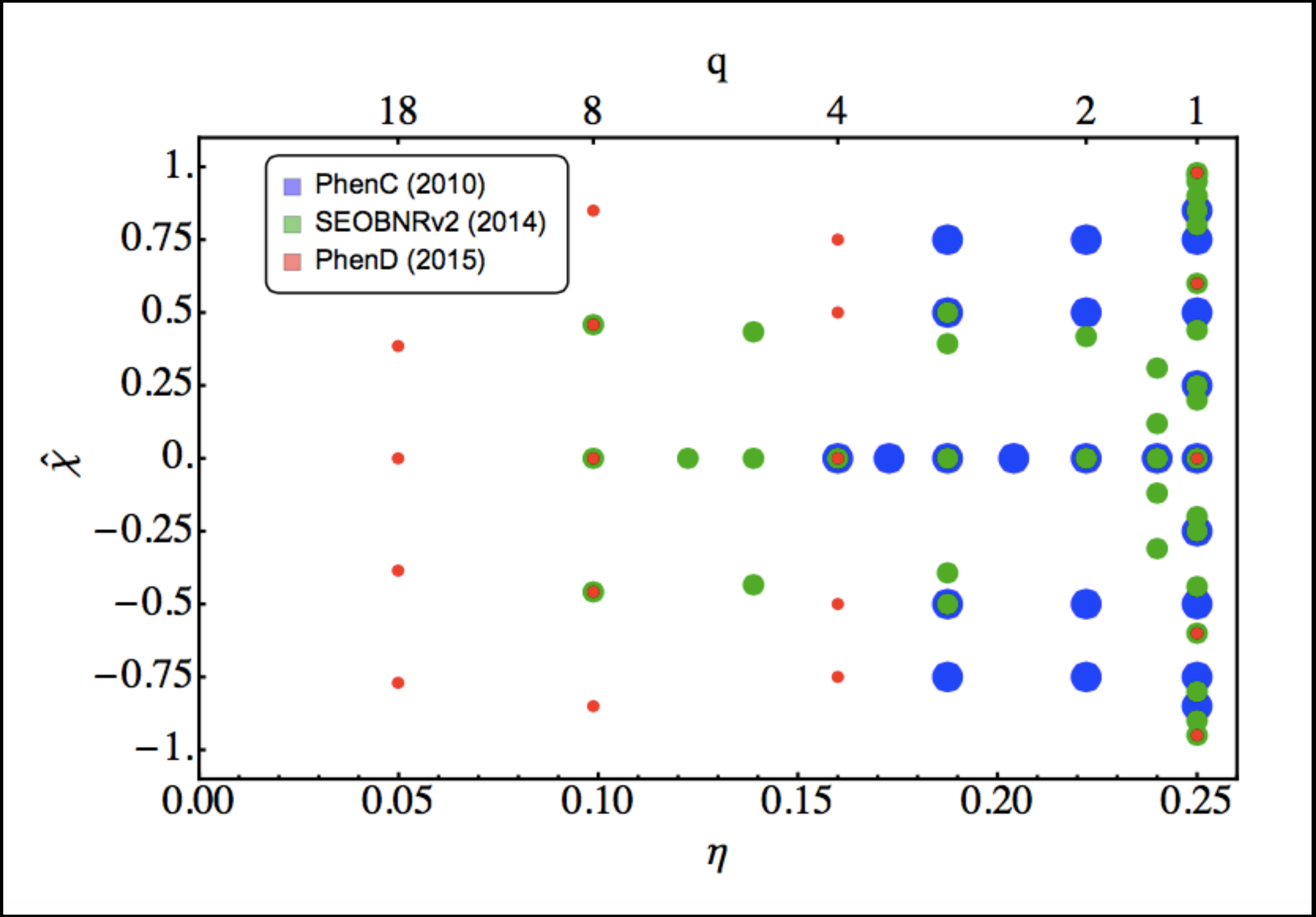


- (a) PN-based ansatz
- (b) phenomenological fit (based on NR behaviour)
- (c) FFT of ringdown waveform (Lorentzian)
- Analytic: fast

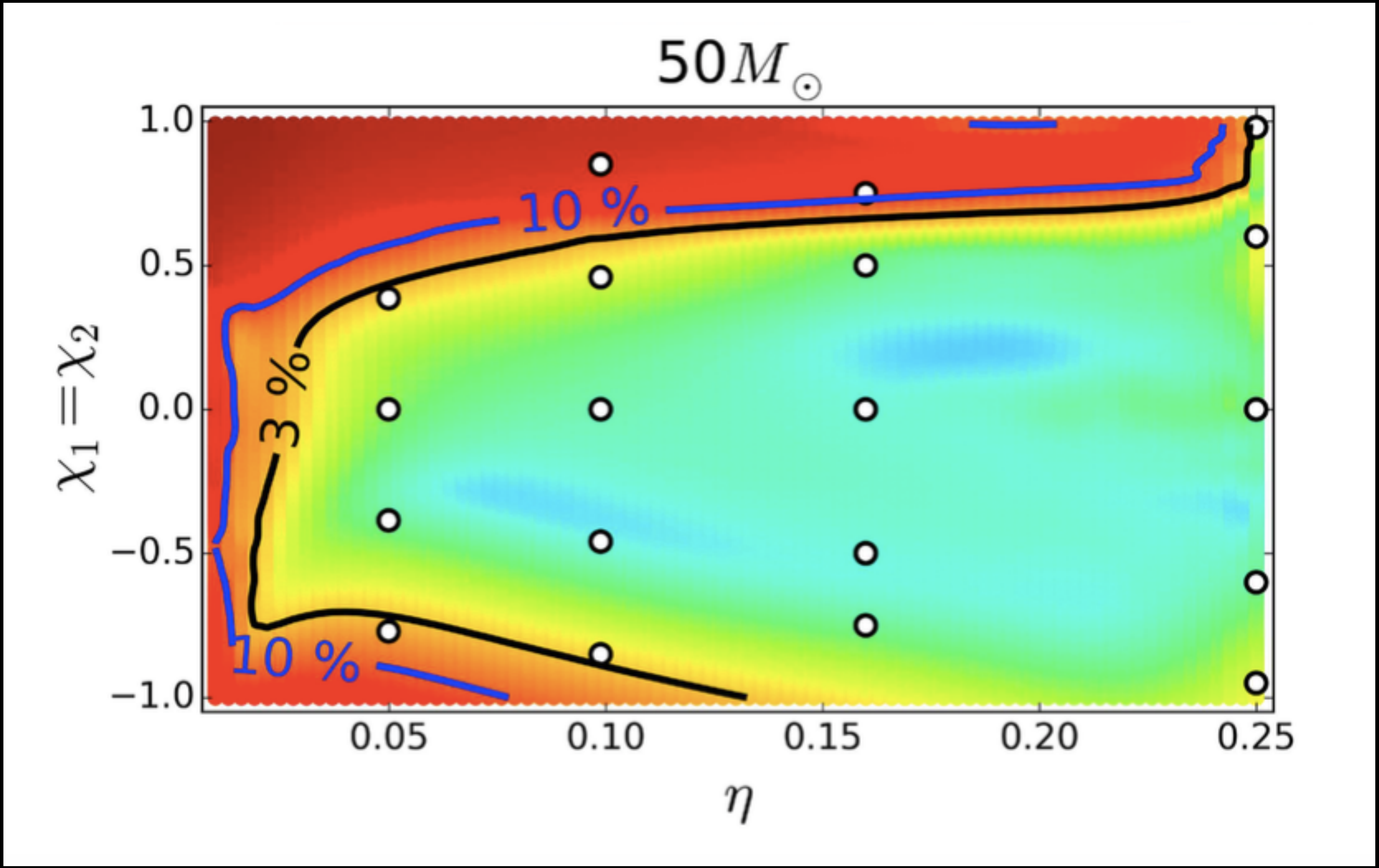
EOB-NR
(time domain)



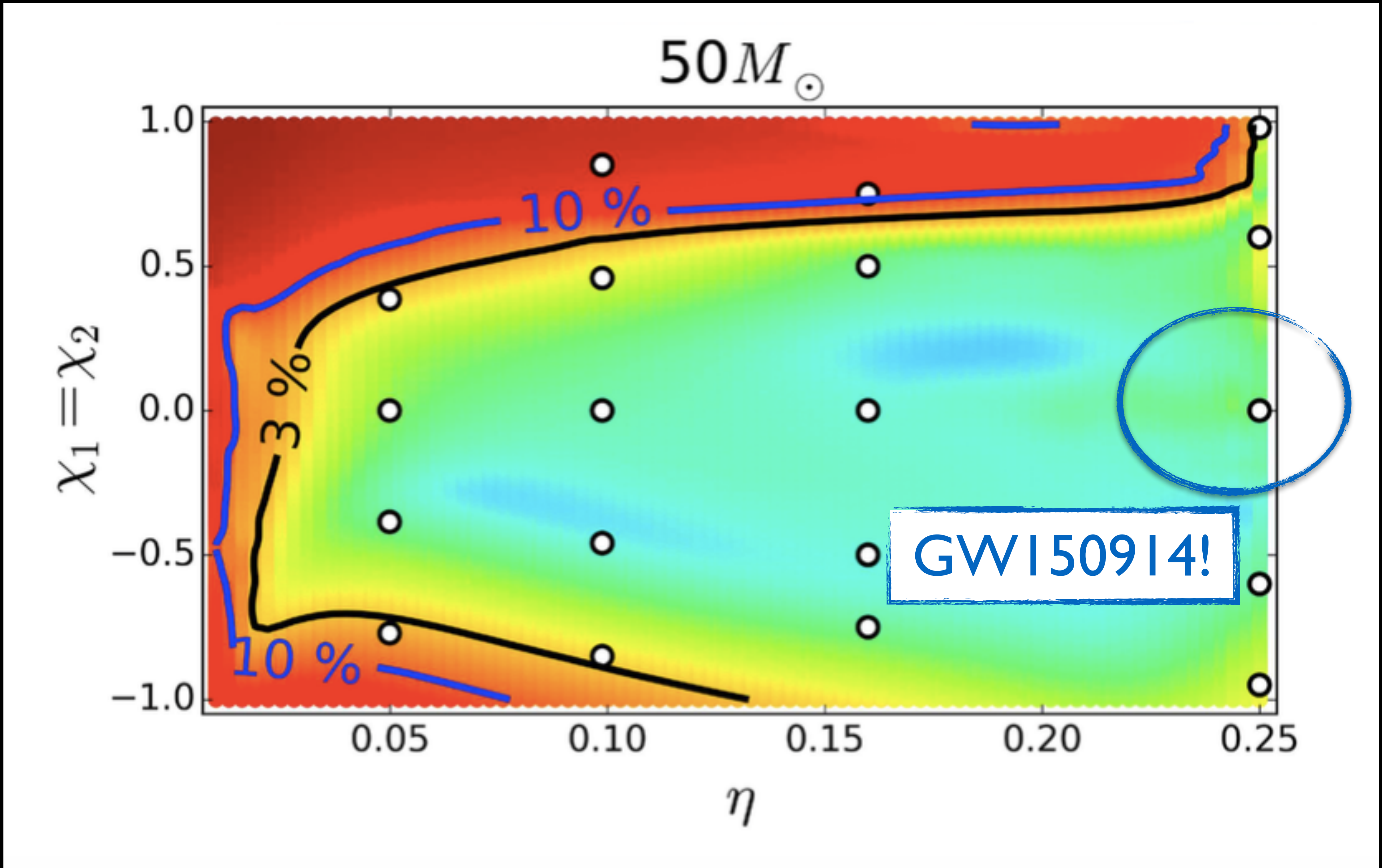
- (a) EOB + terms tuned to NR waveforms
- (b) Smooth transition to ringdown
- Includes both spins
- Numerically solve ODEs: slow
- Speed-up: Reduced-order models



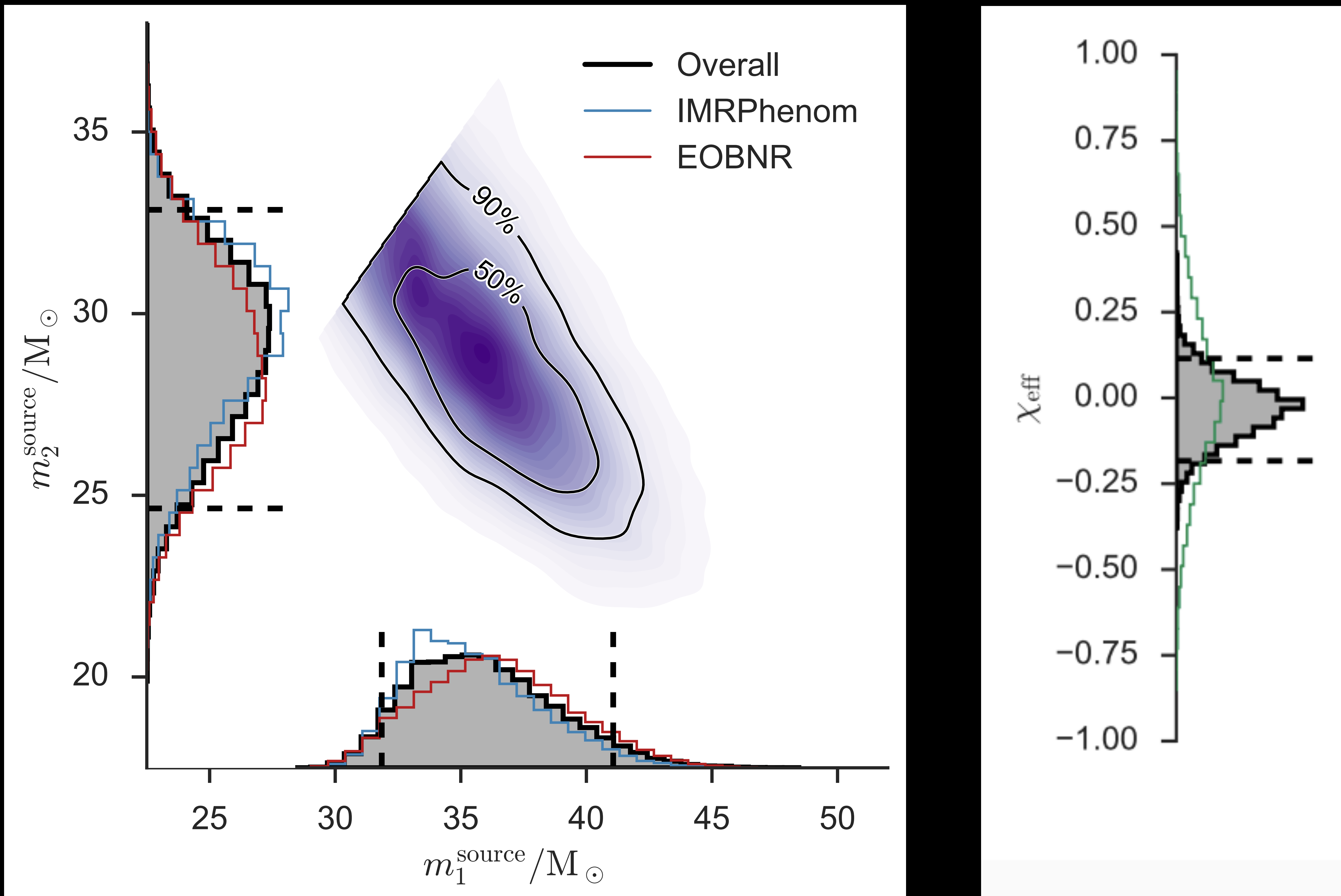
[Khan, *et. al* (2016)]



[Khan, *et. al* (2016)]

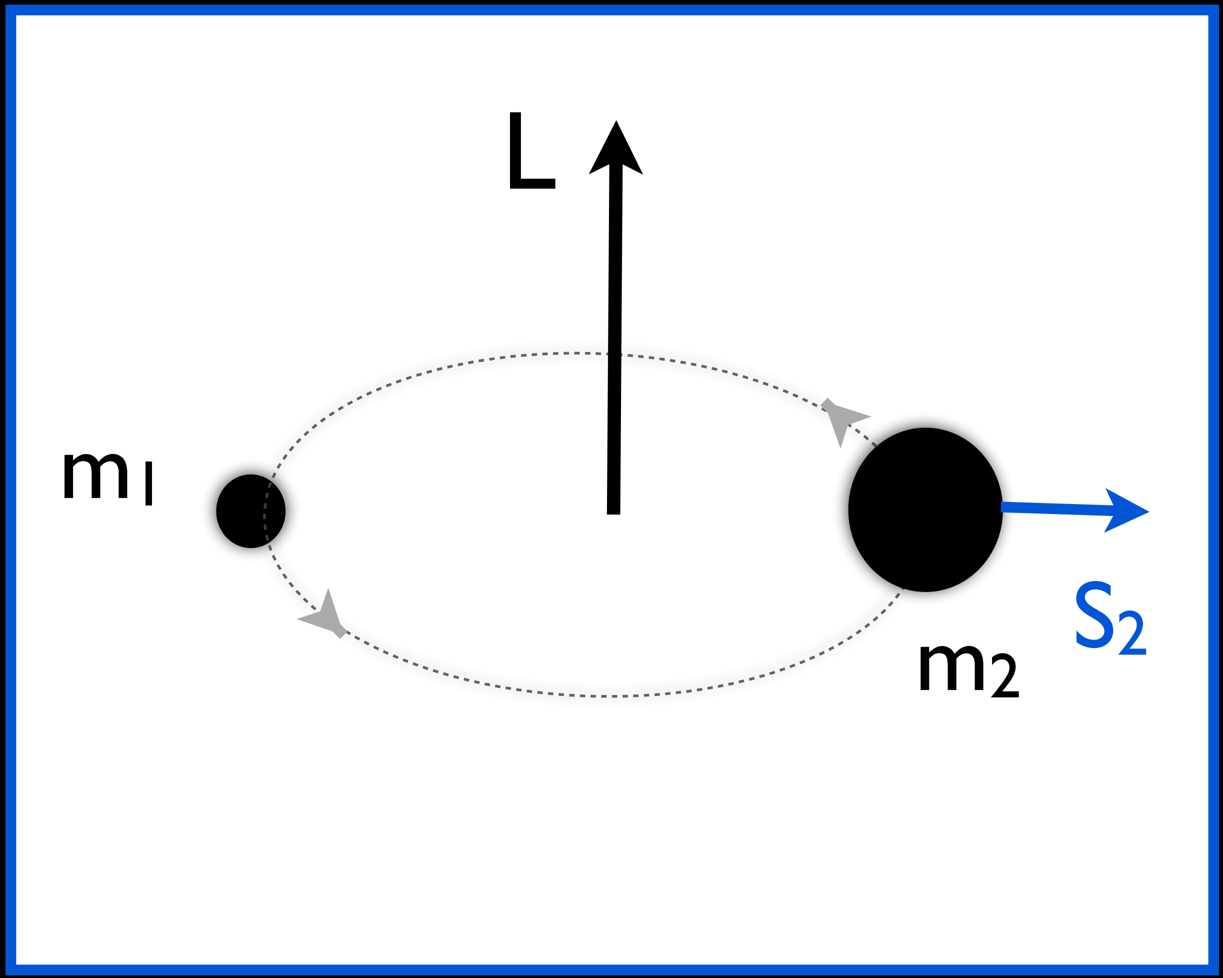


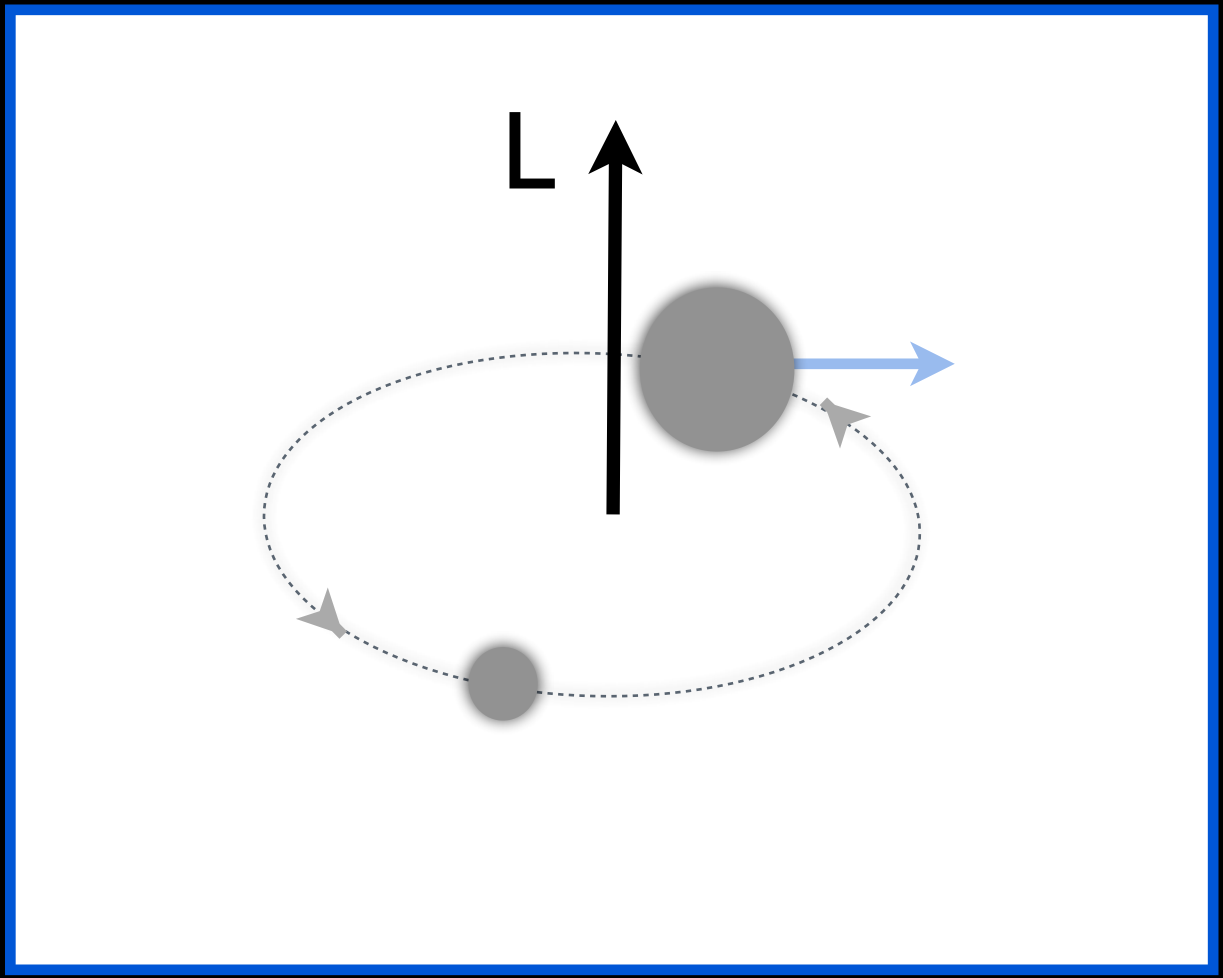
[Khan, *et. al* (2016)]



Overall
IMRPhenom
EOBNR
90%
50%
20
25
30
35
$m_2^{\rm source}/{\rm M}_\odot$
25
30
35
40
45
50
$m_1^{\rm source}/{\rm M}_\odot$
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
$\chi_{\rm eff}$

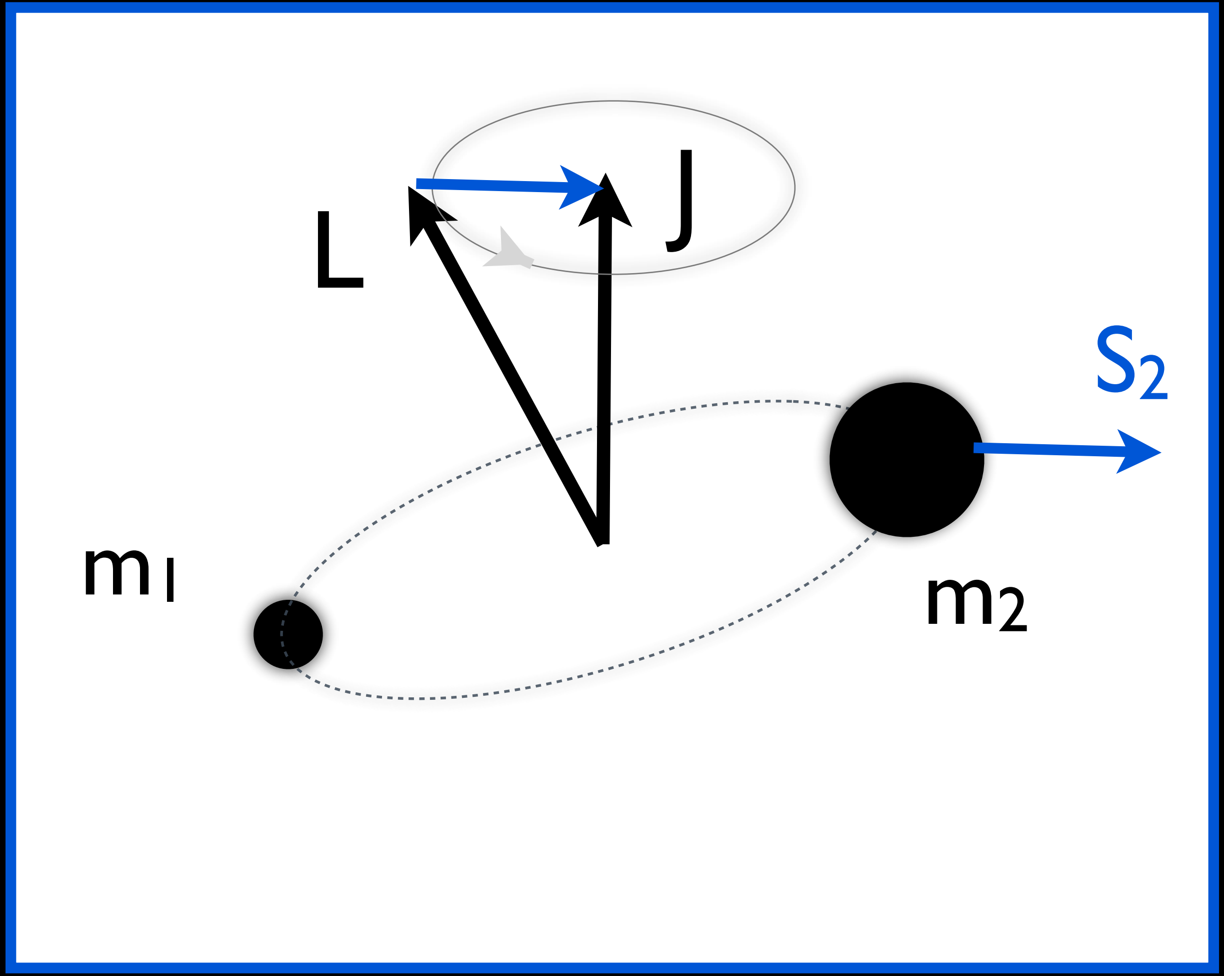
Orbital precession

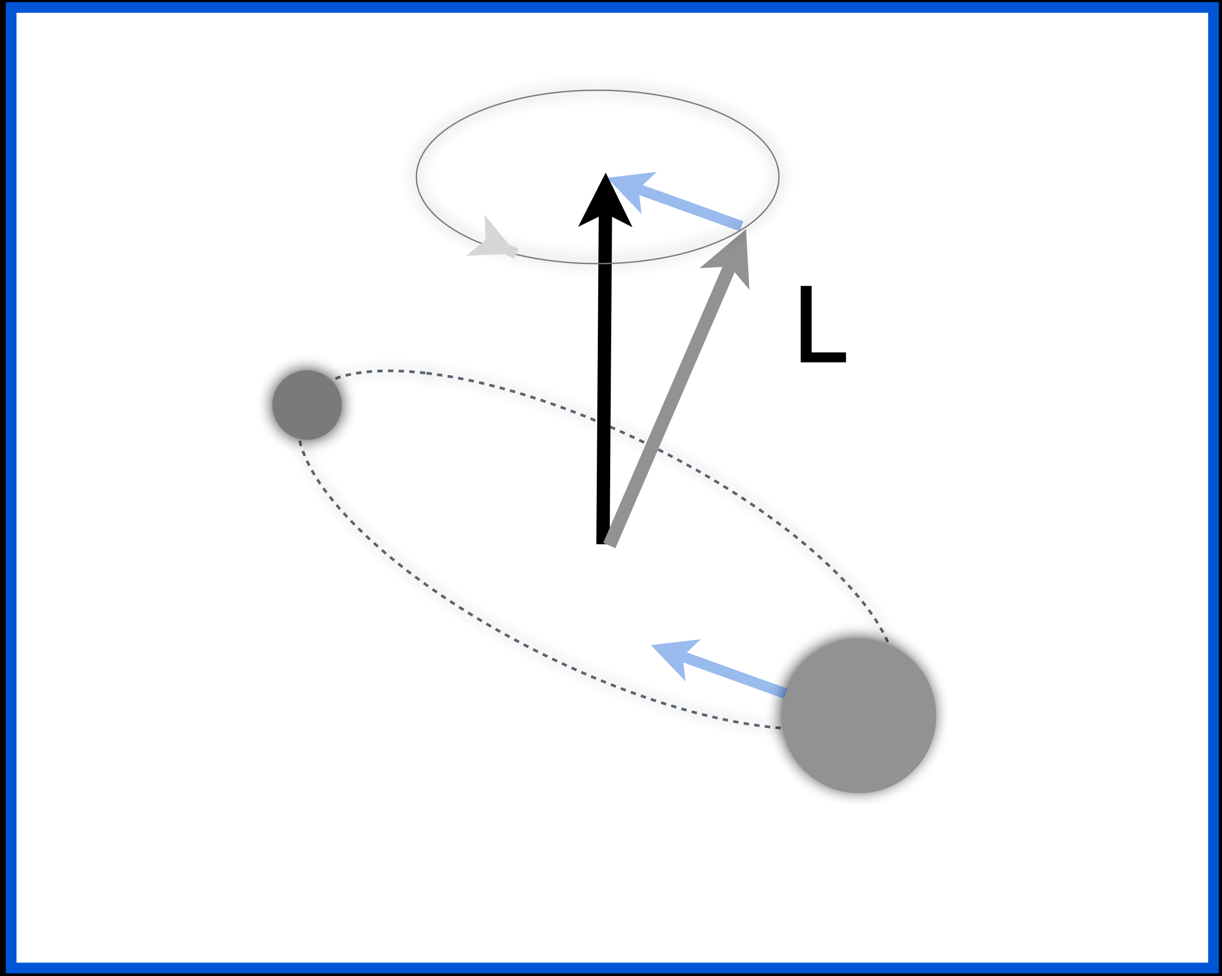




Newtonian gravity:
L, S_1, S_2 remain fixed

Orbital precession

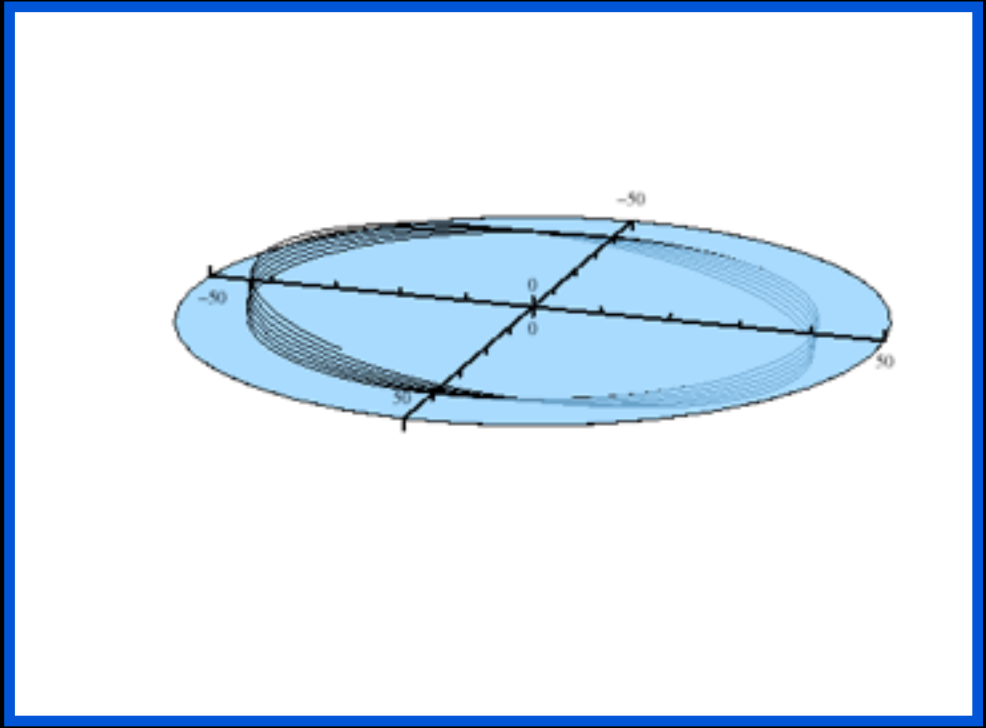




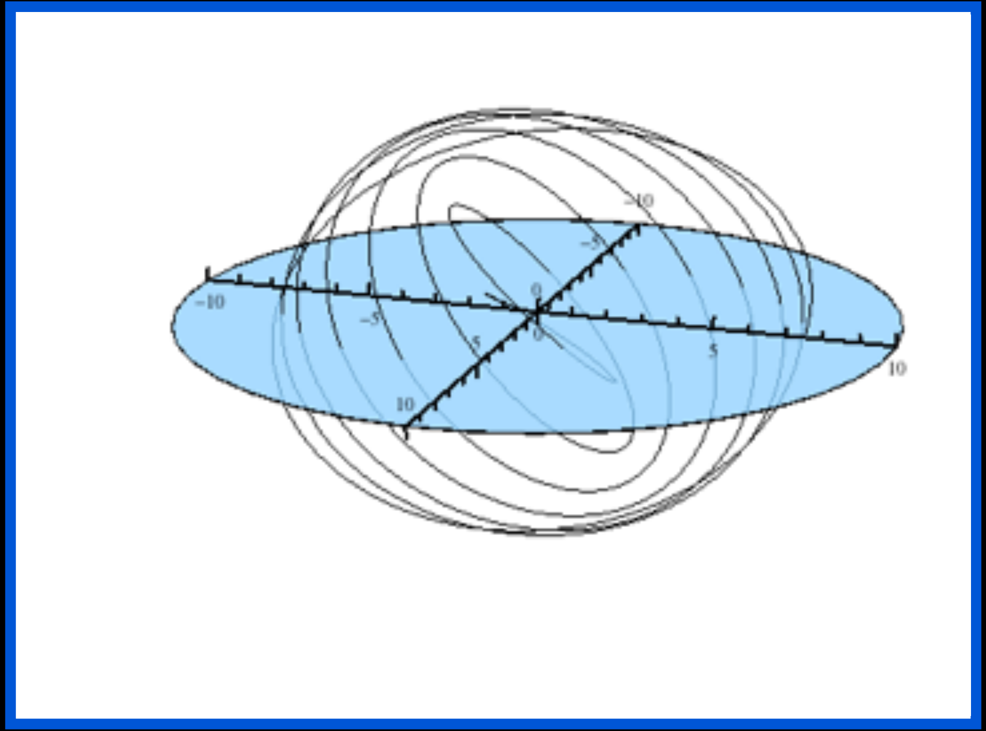
General relativity

(L, S_1, S_2) precess around J

Precessional dynamics



Large separation



Merger

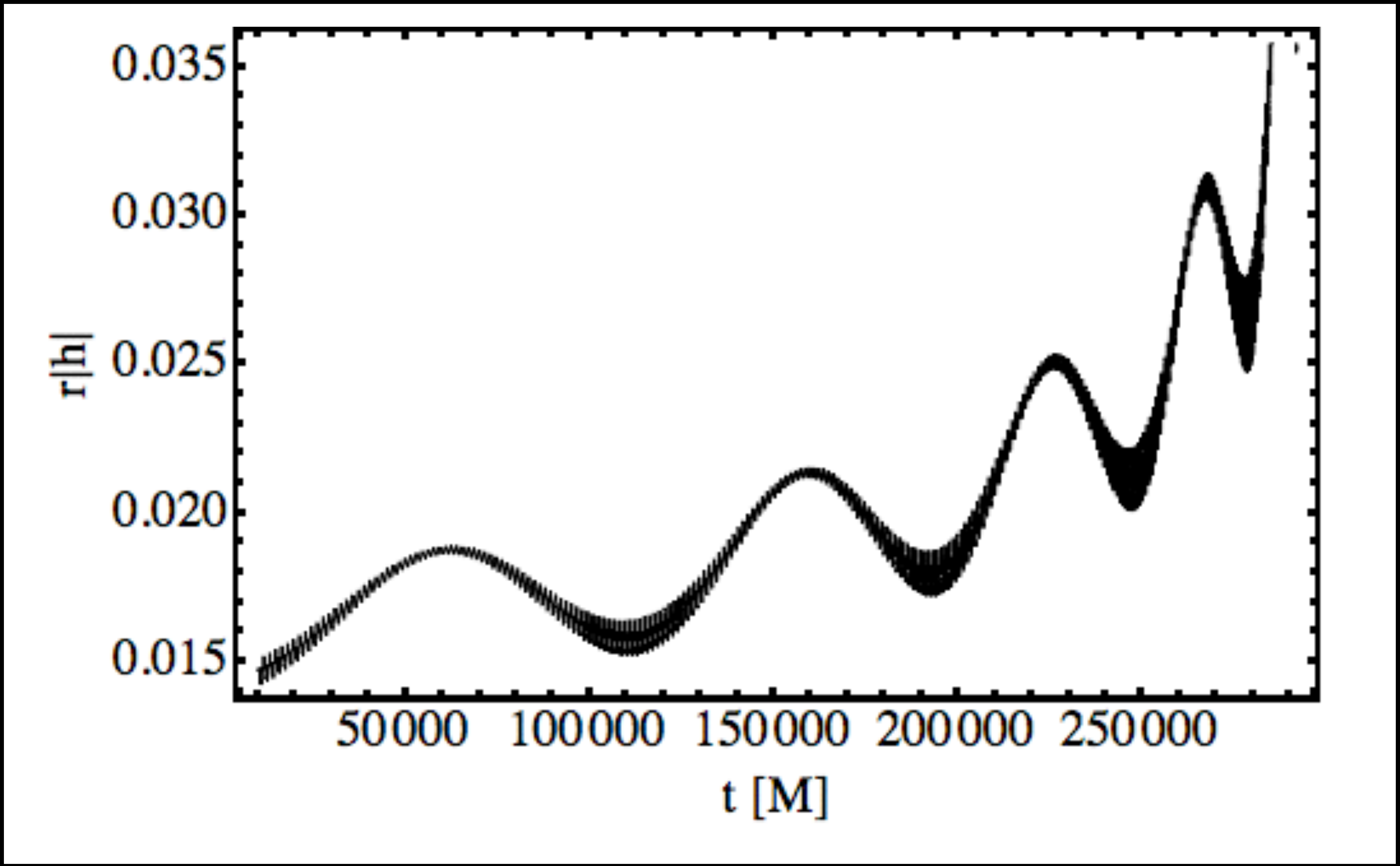
Aside: modelling precession

Precessing waveform =

(non-precessing waveform)

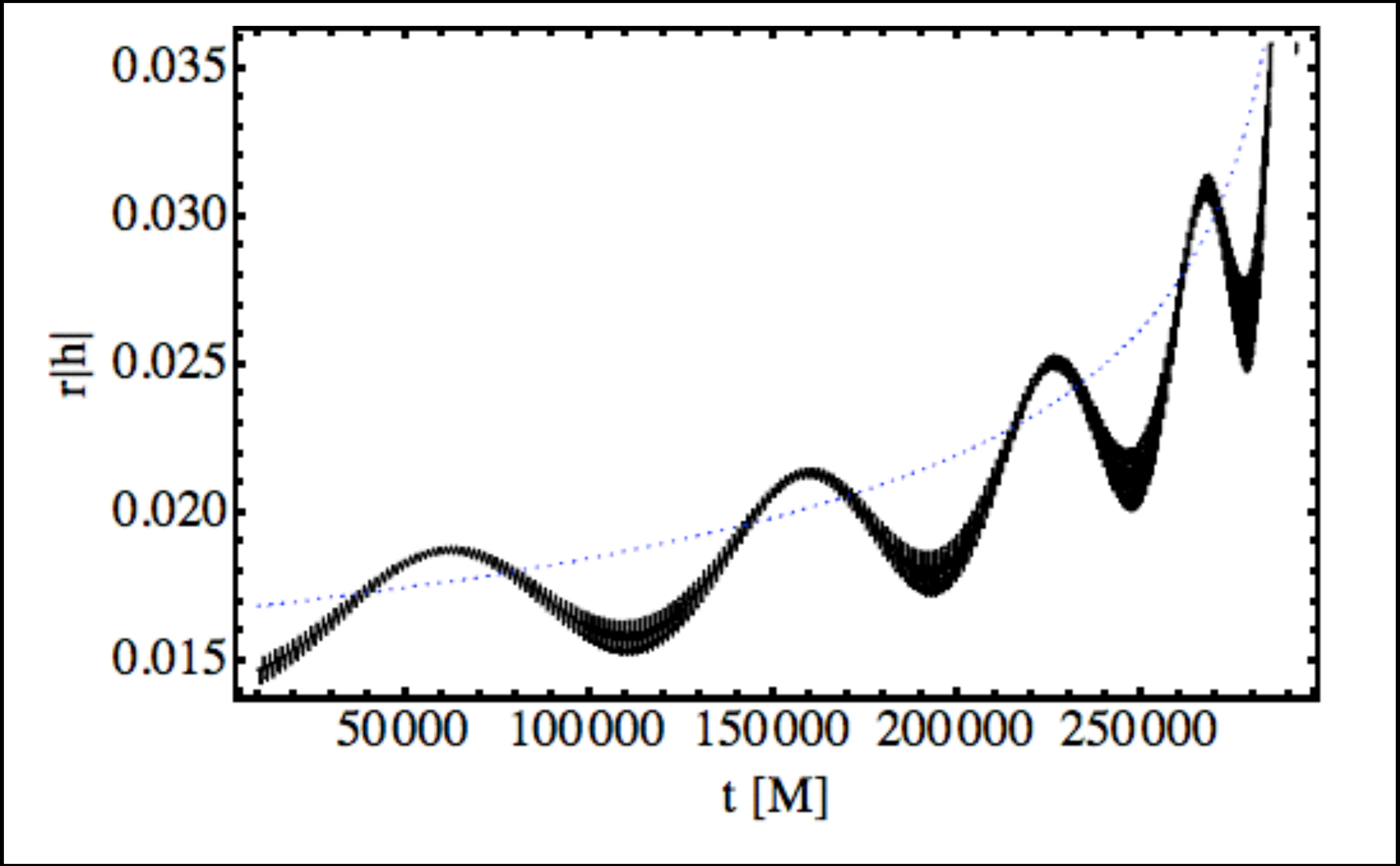
x (time dependent rotation)

(q=3 precessing binary, inclination 2.8 rad)



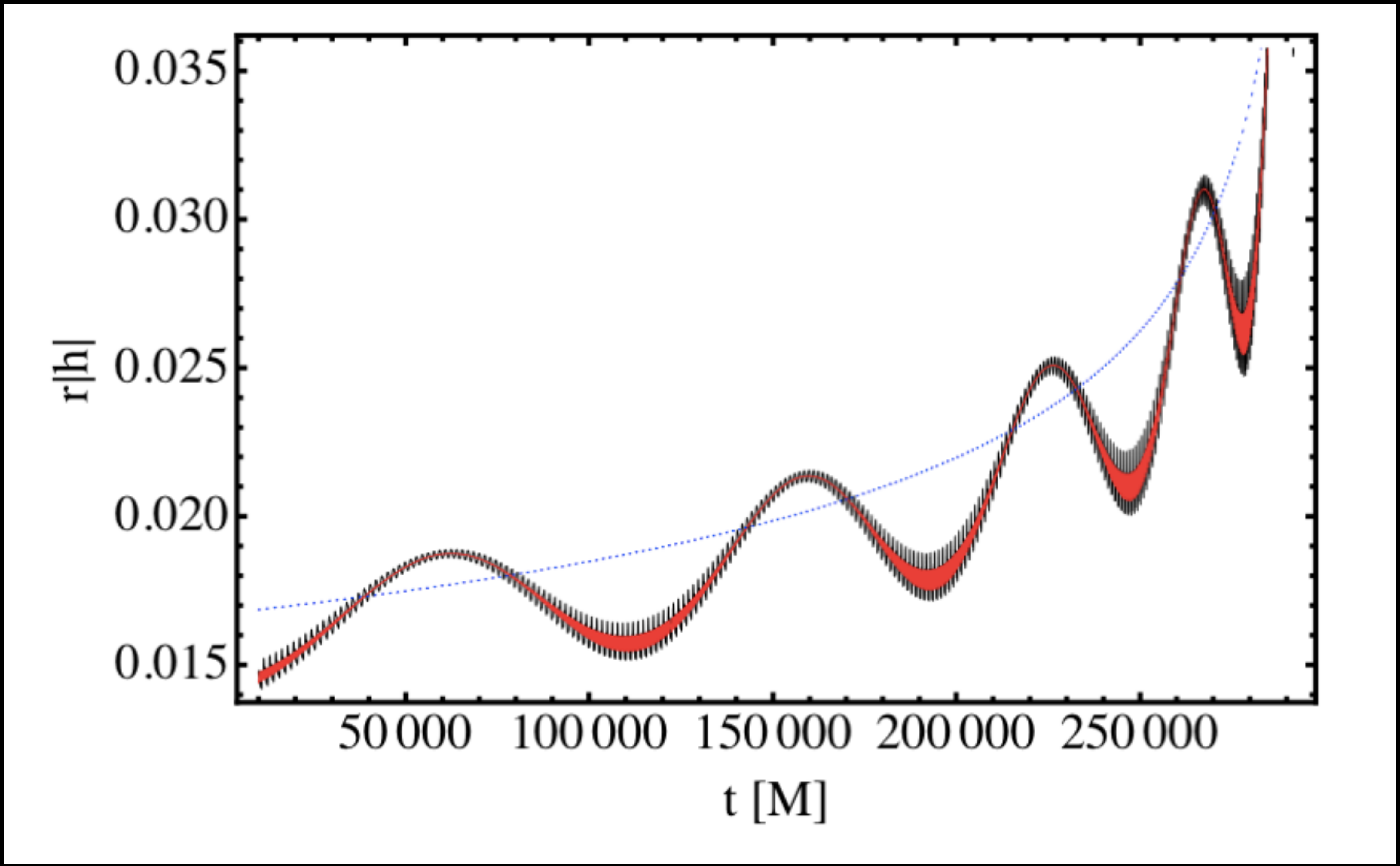
[Schmidt, Hannam, Husa (2012)]

(q=3 precessing binary, inclination 2.8 rad)



[Schmidt, Hannam, Husa (2012)]

(q=3 precessing binary, inclination 2.8 rad)



[Schmidt, Hannam, Husa (2012)]

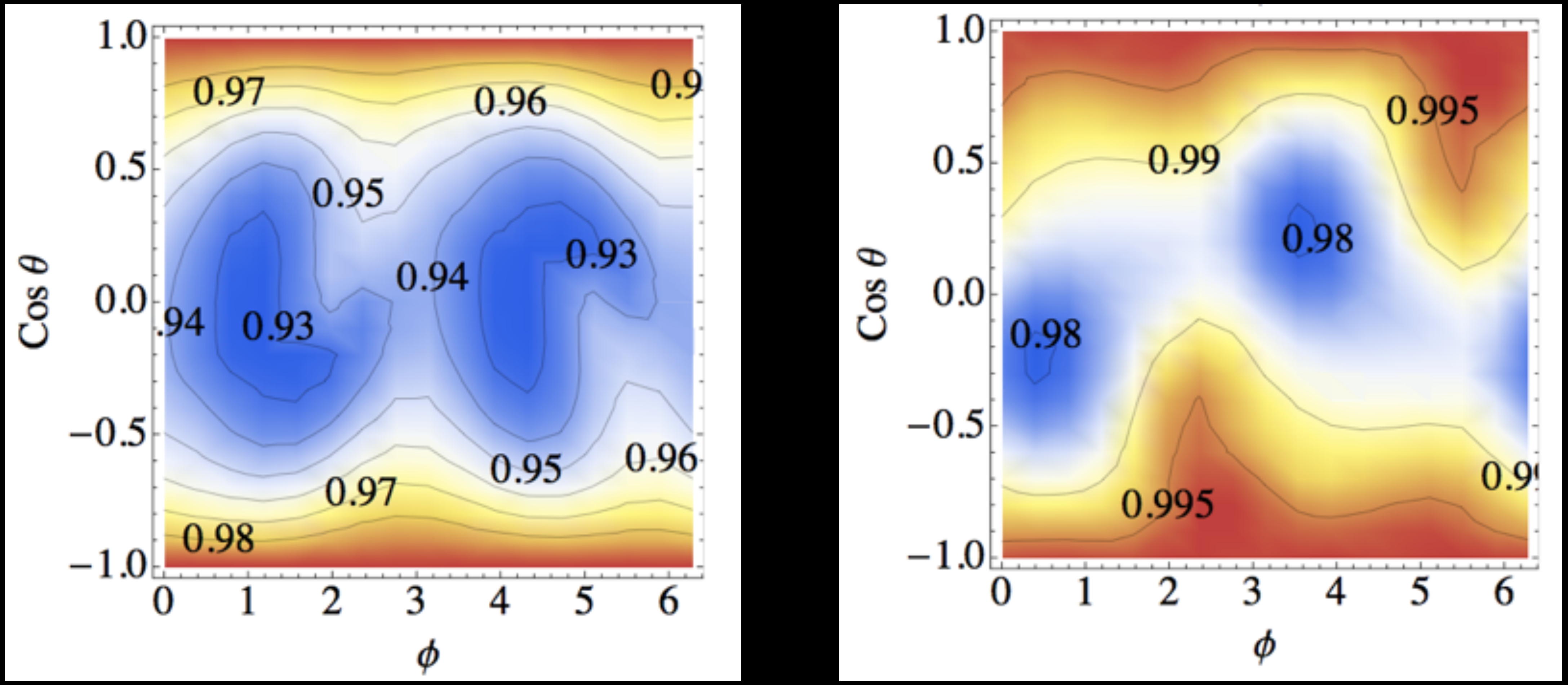
For non-precessing binaries spin effects dominated by only *one* key spin parameter

Does something similar happen for precession?
i.e., can we replace
the four in-plane spin components
with *one* "precession spin"?

Yes!

PhenomP

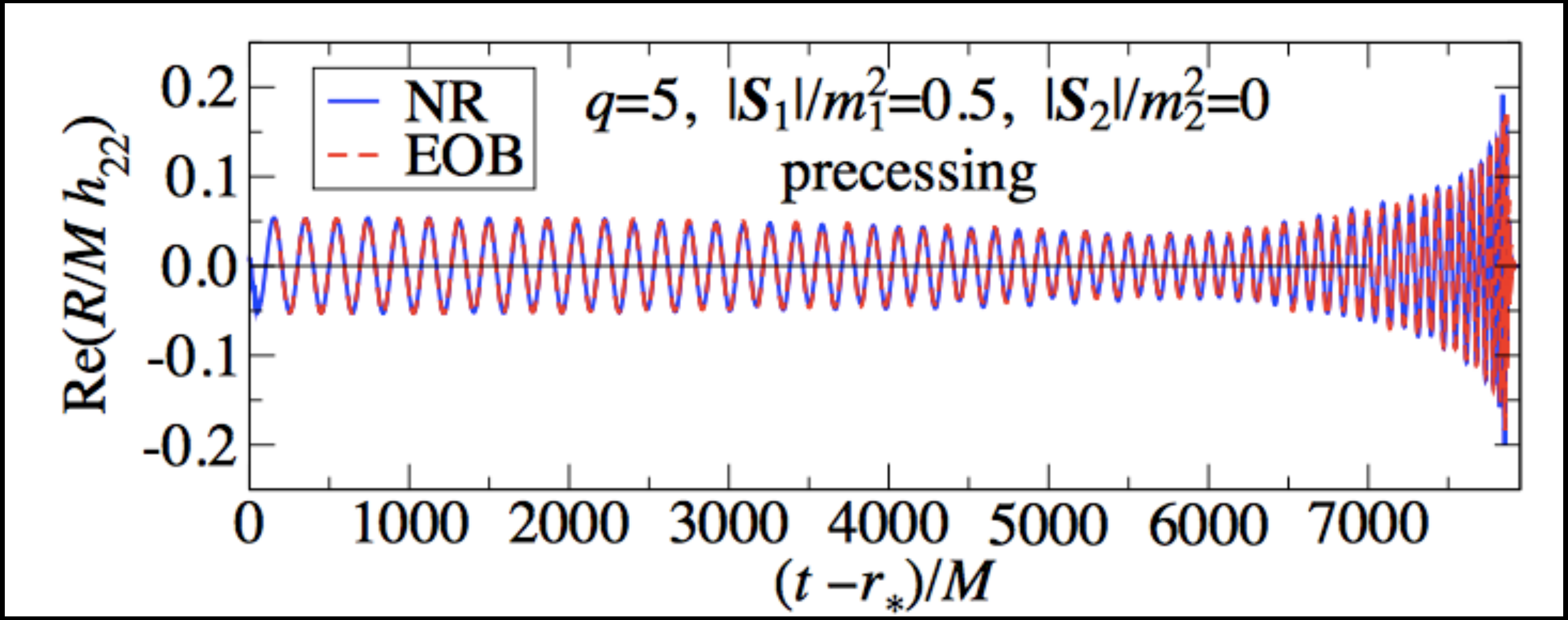
- Non-precessing model: PhenomD
- Twist with (analytic) PN precession angles
- Approximation: use PN angles through ringdown.



[Hannam, et al. (2014)]

SEOBNRv3

- Non-precessing model: inspiral part of SEOBNRv2
- Twist with solution of precessing-EOB dynamics
- Attach ringdown
- Includes all 6 spin components



[Pan, et al. (2014), Taracchini, et. al. (2014)]

Neither precessing model
is tuned to
precessing NR simulations!

Orientation dependence

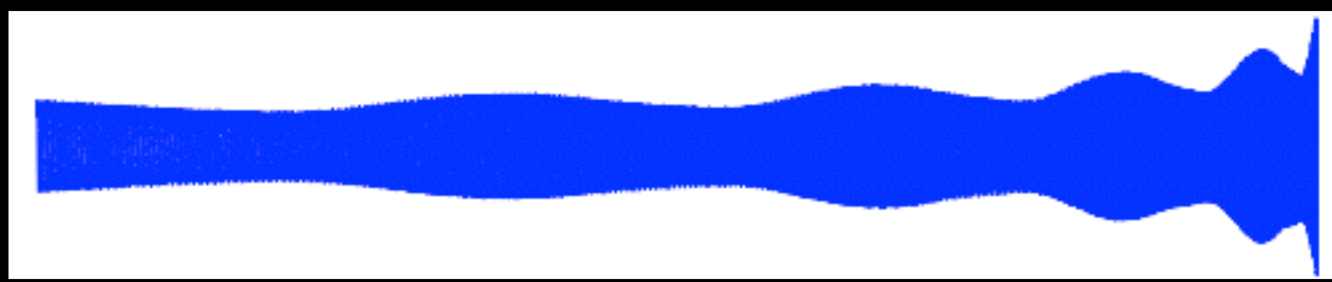
$q=3$, $|S_2| = 0.75$ (in plane)

Observer aligned with J

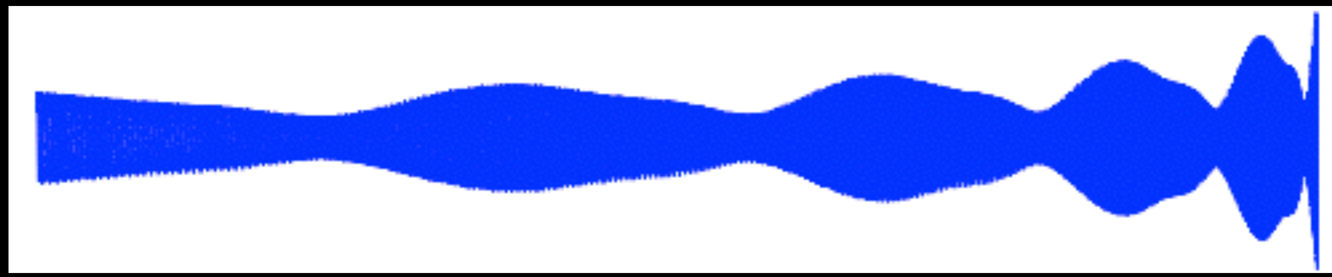
Orientation dependence

q=3, $|S_2|$ = 0.75 (in plane)

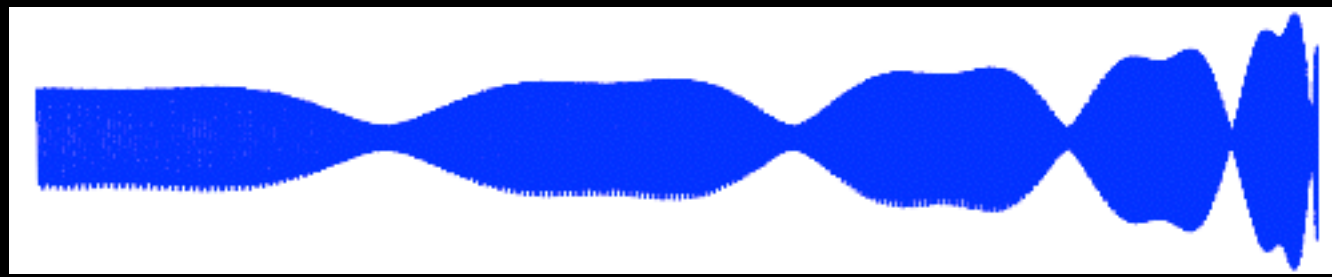
Observer aligned with *J*



Observer inclined $\pi/6$ to *J*

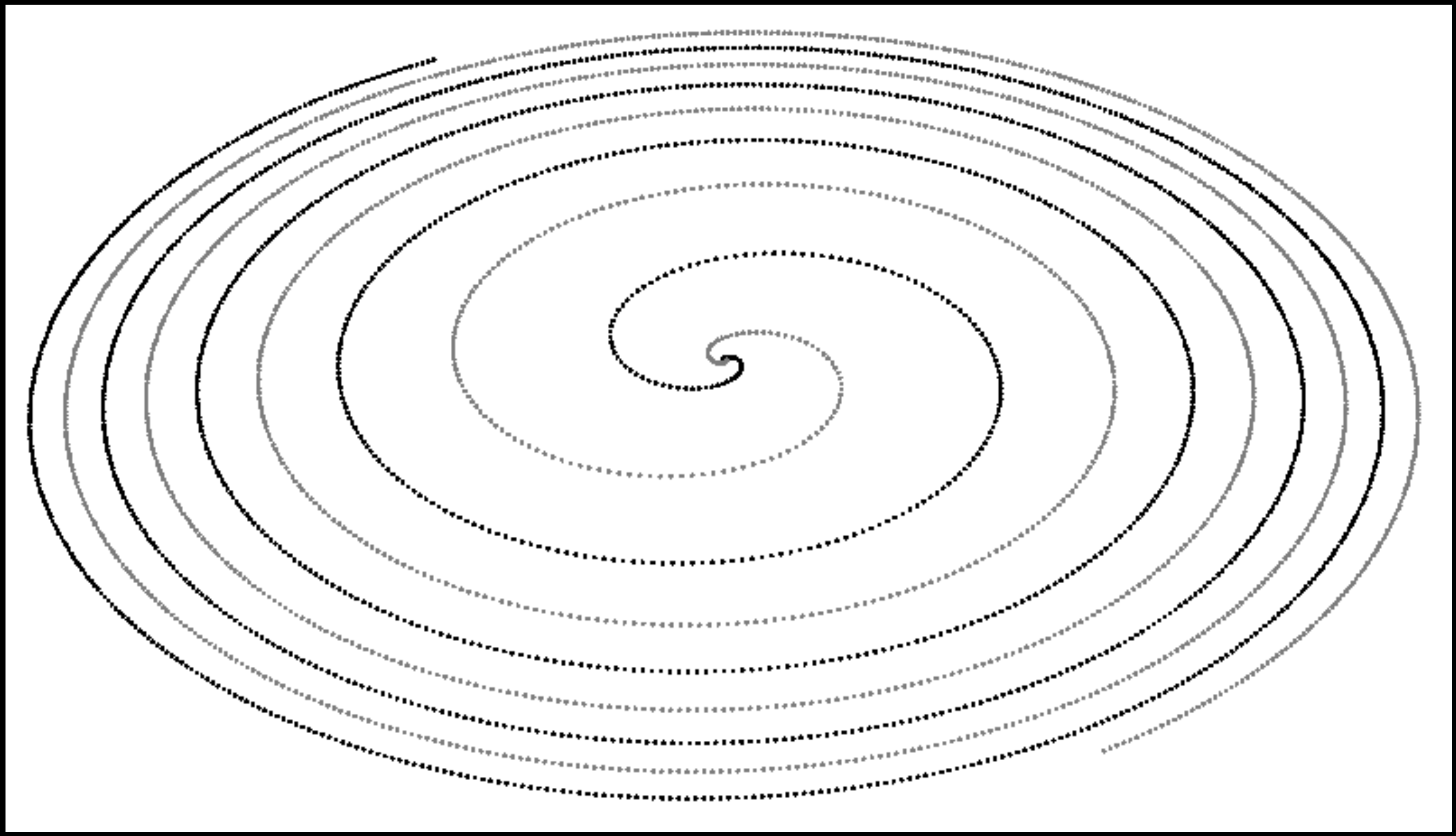


Observer inclined $\pi/3$ to *J*

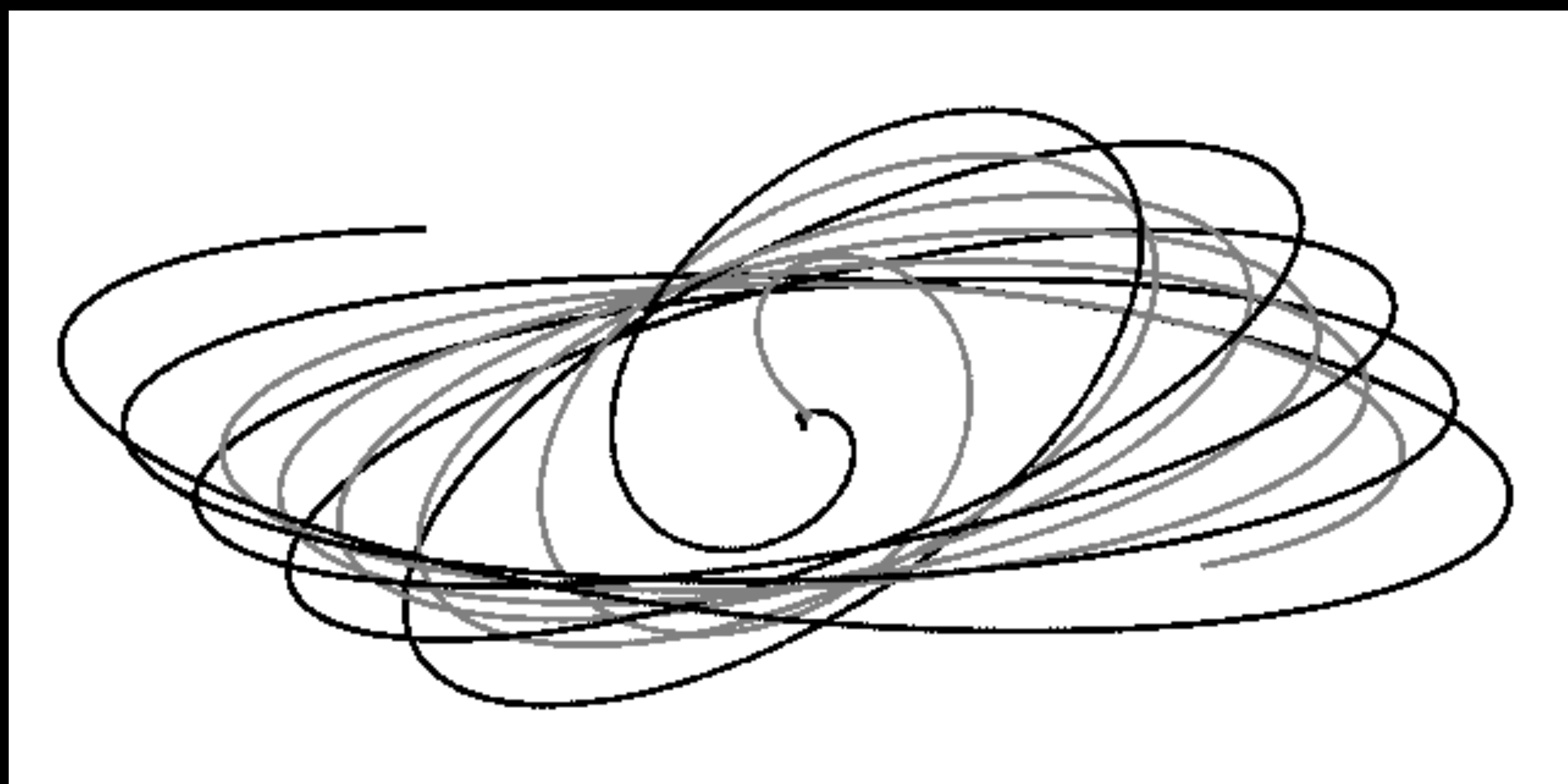


Observer inclined $\pi/2$ to *J*

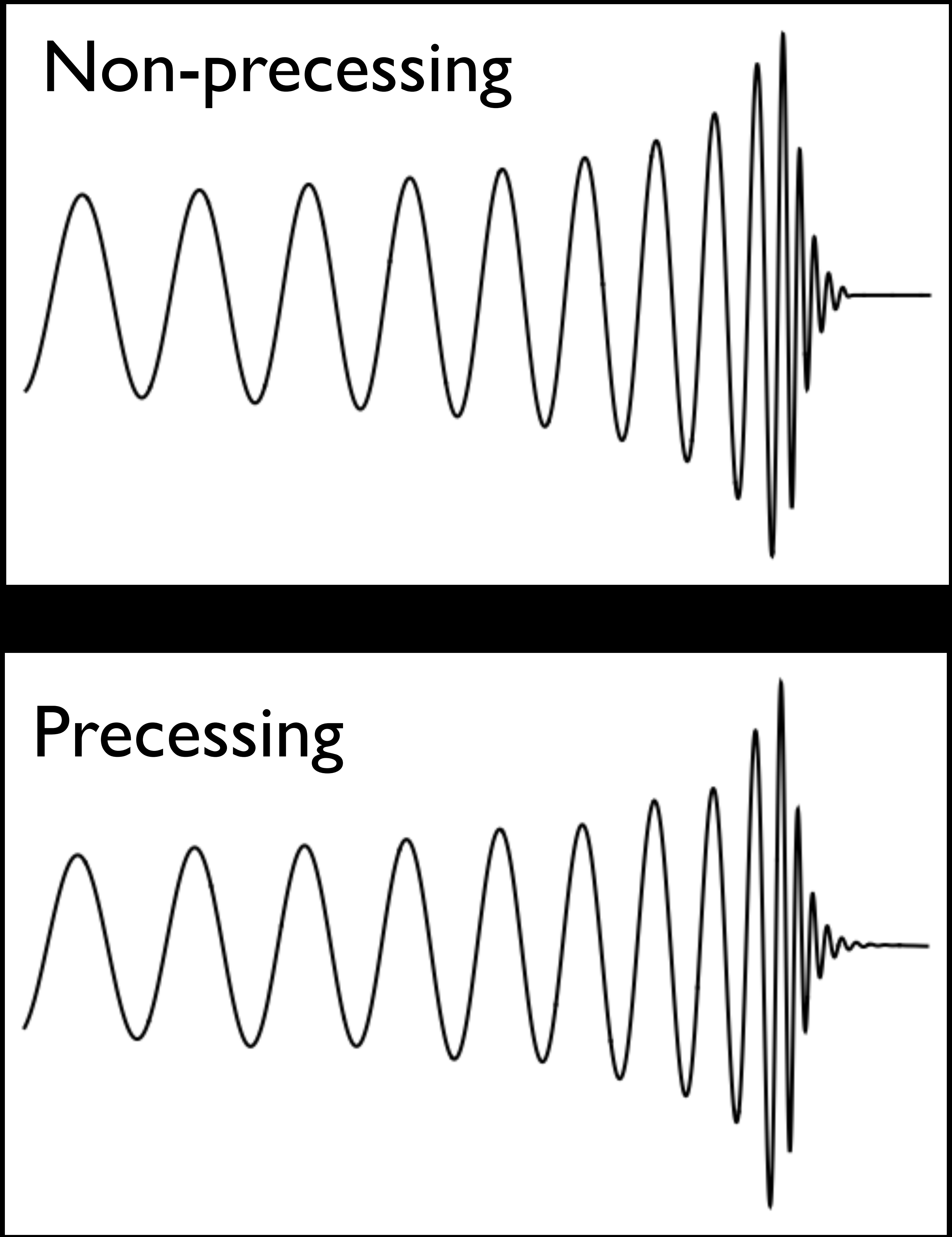
Equal-mass nonspinning BBH consistent with GW150914



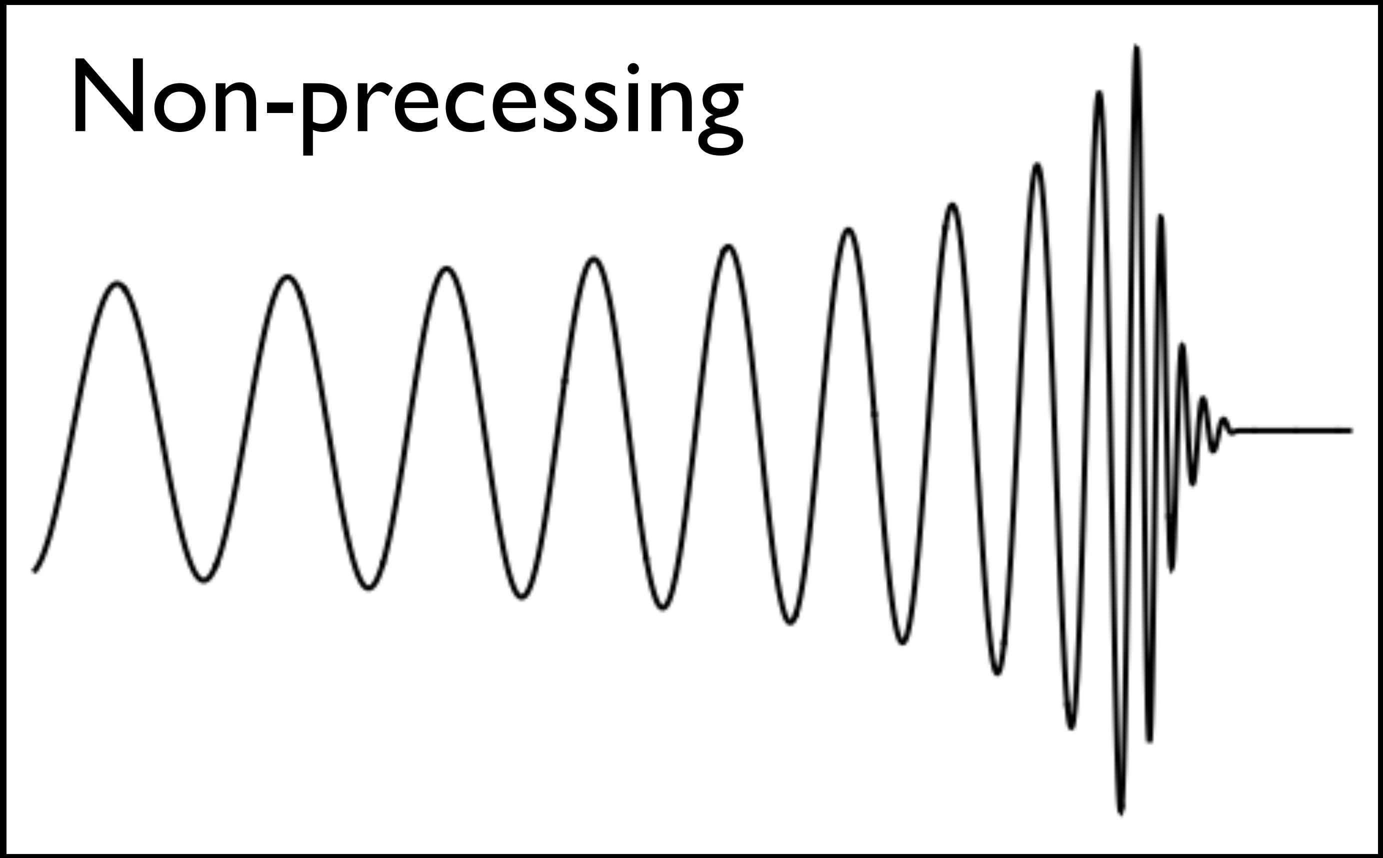
Unequal-mass precessing BBH consistent with GW150914

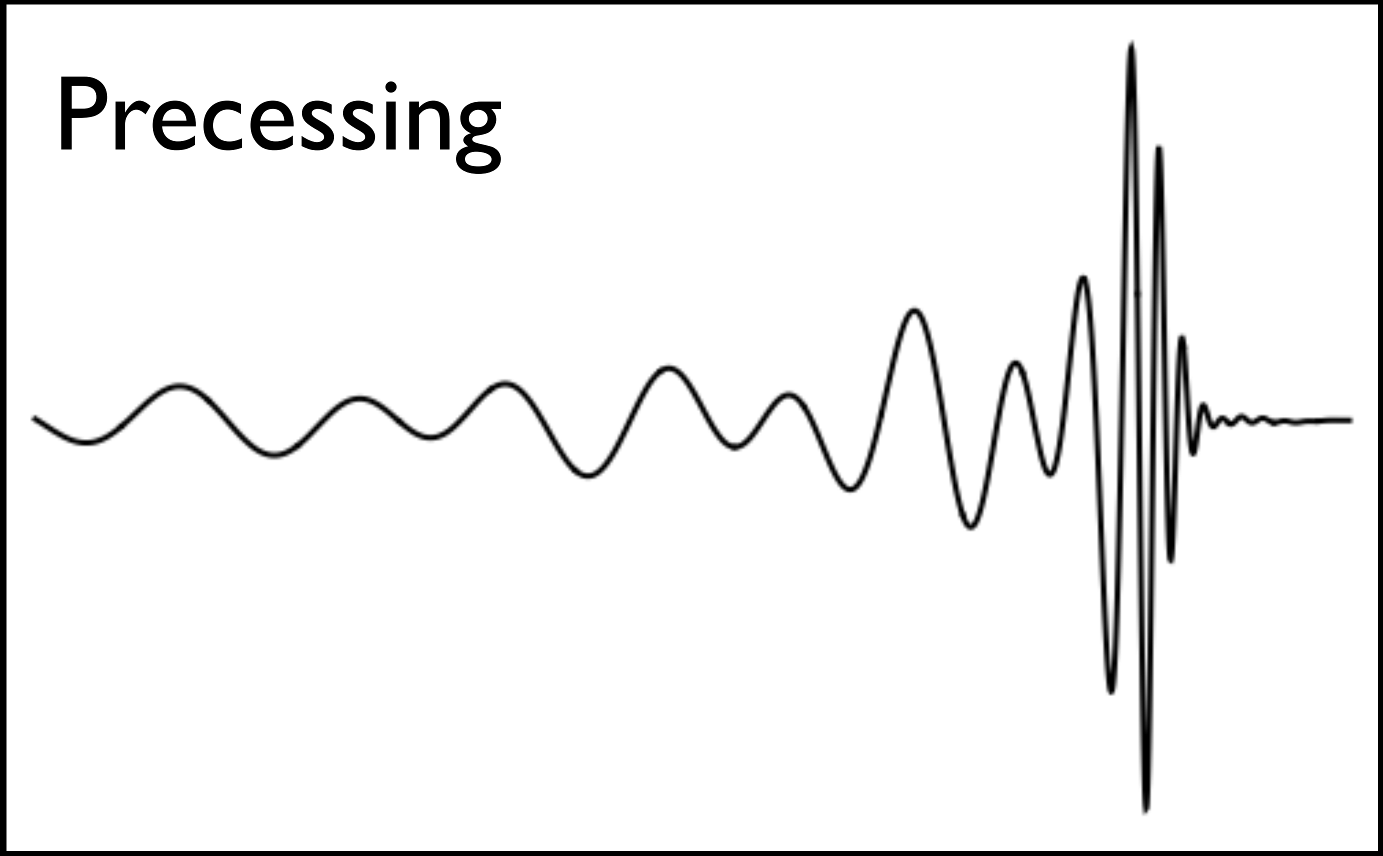


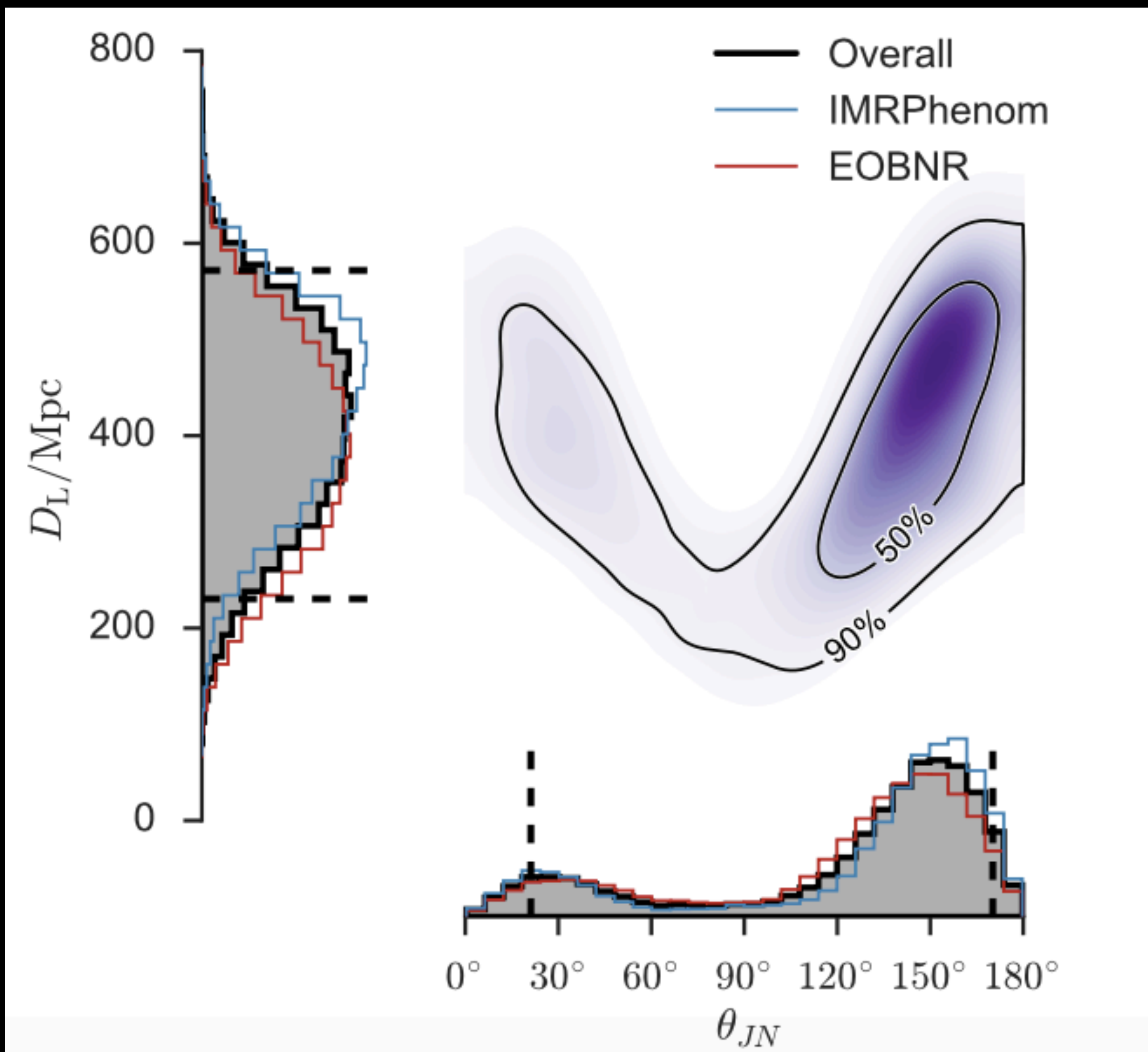
"Face-on" to the source

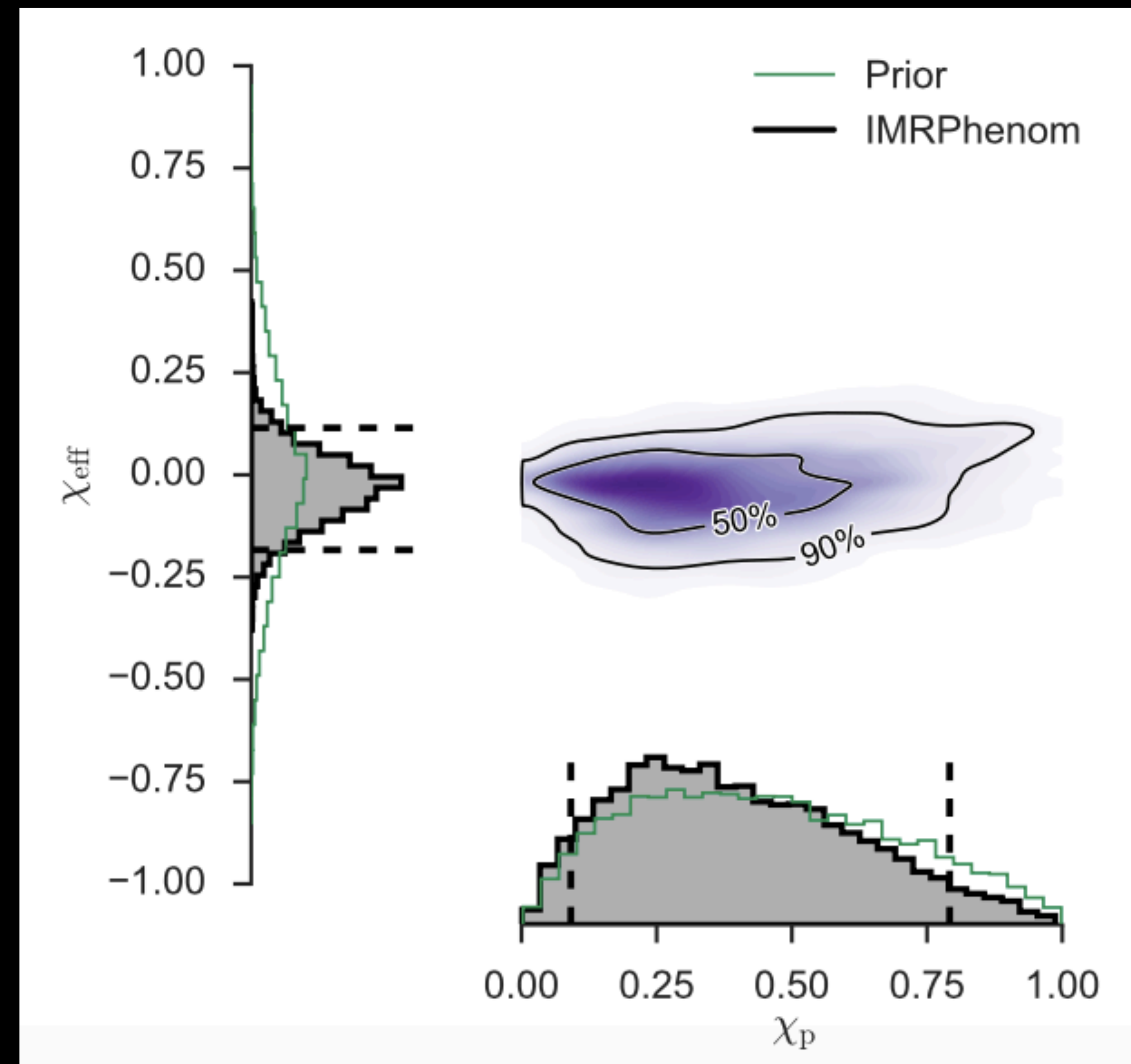


"Edge-on" to the source

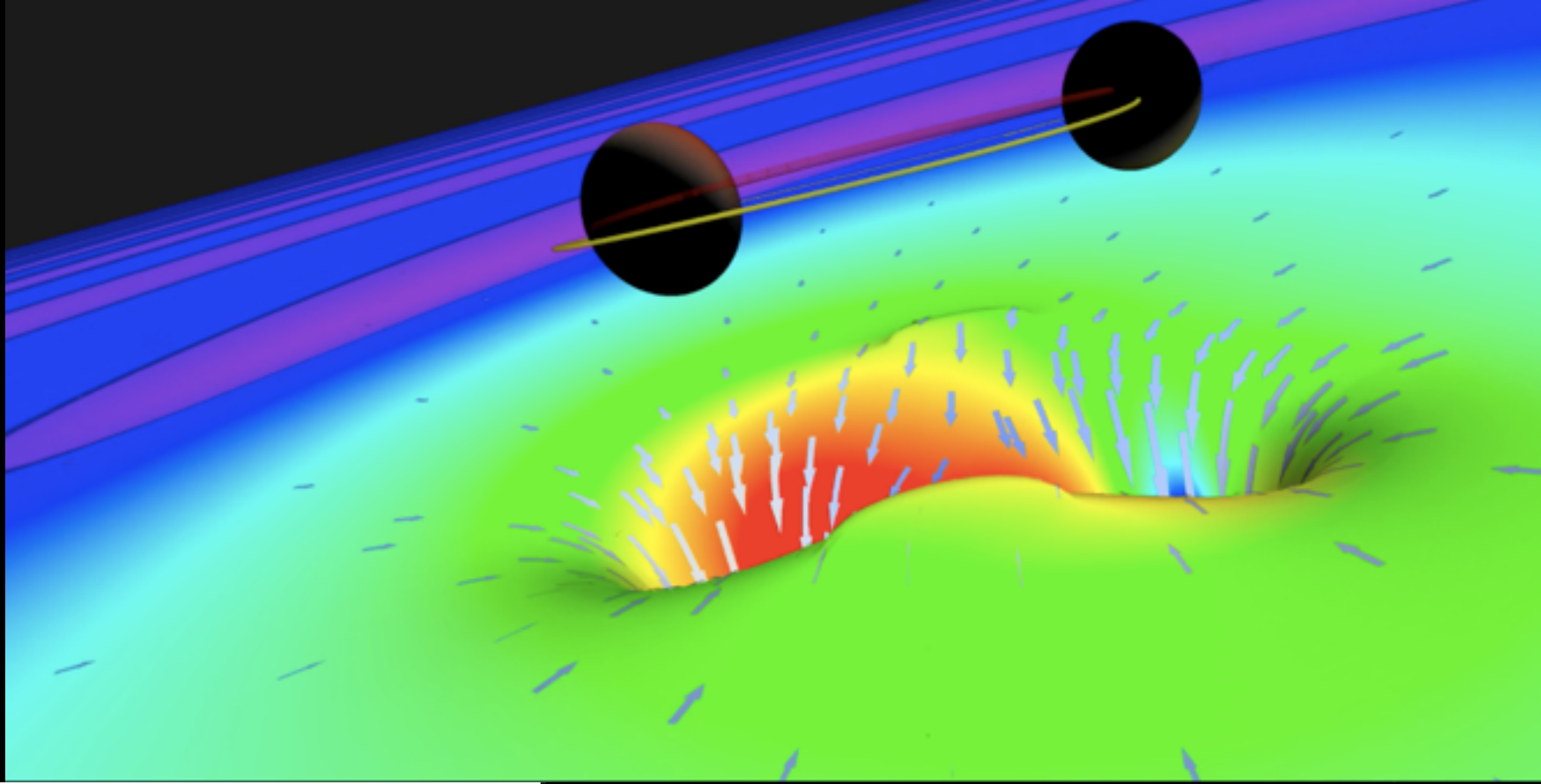


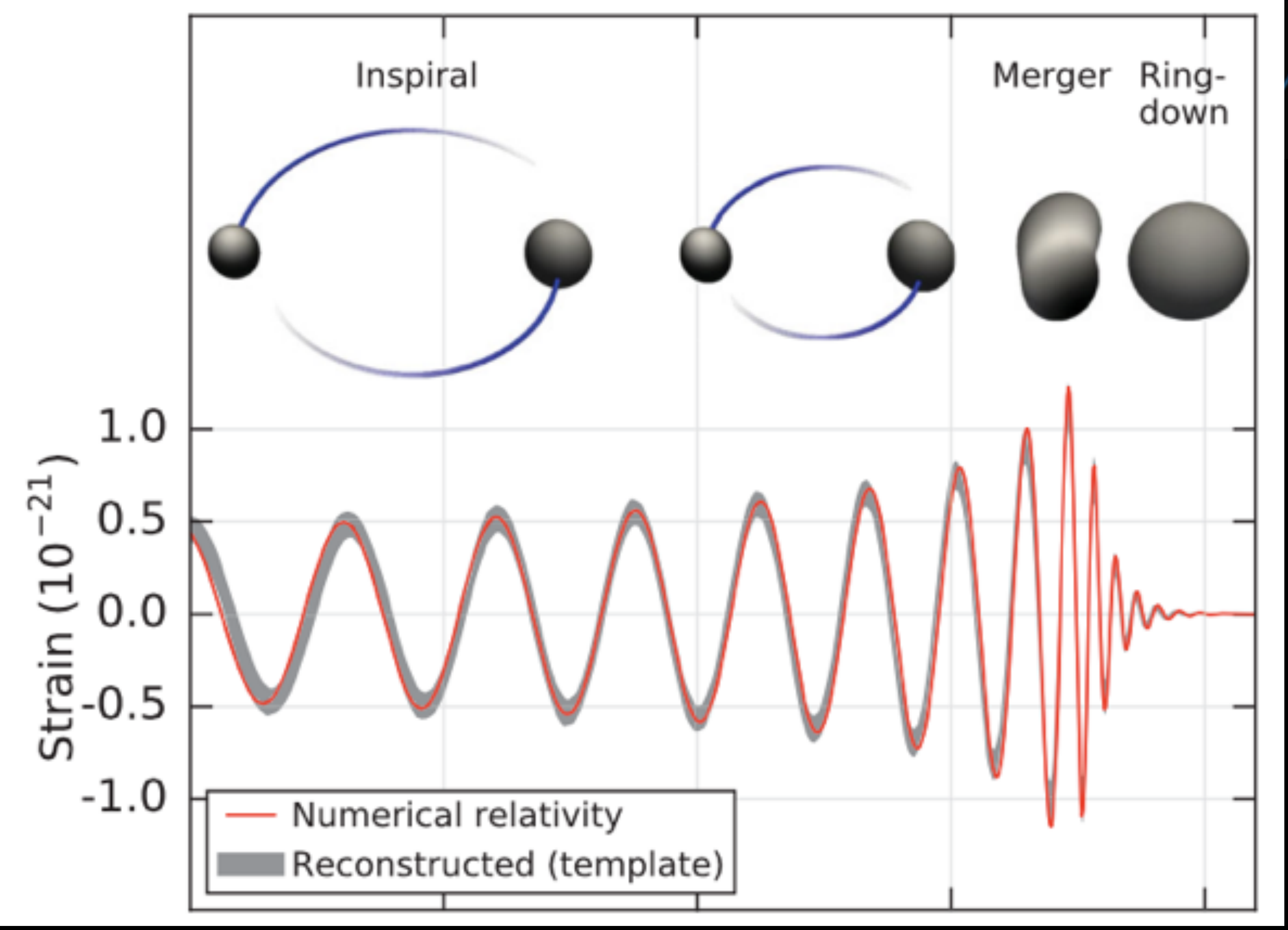



Overall
IMRPhenom
EOBNR
800
600
400
200
0
D_L/Mpc
50%
90%
0° 30° 60° 90° 120° 150° 180°
θ_{JN}


Prior
IMRPhenom
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
$\chi_{\rm eff}$
50%
90%
0.00 0.25 0.50 0.75 1.00
$\chi_{\rm p}$

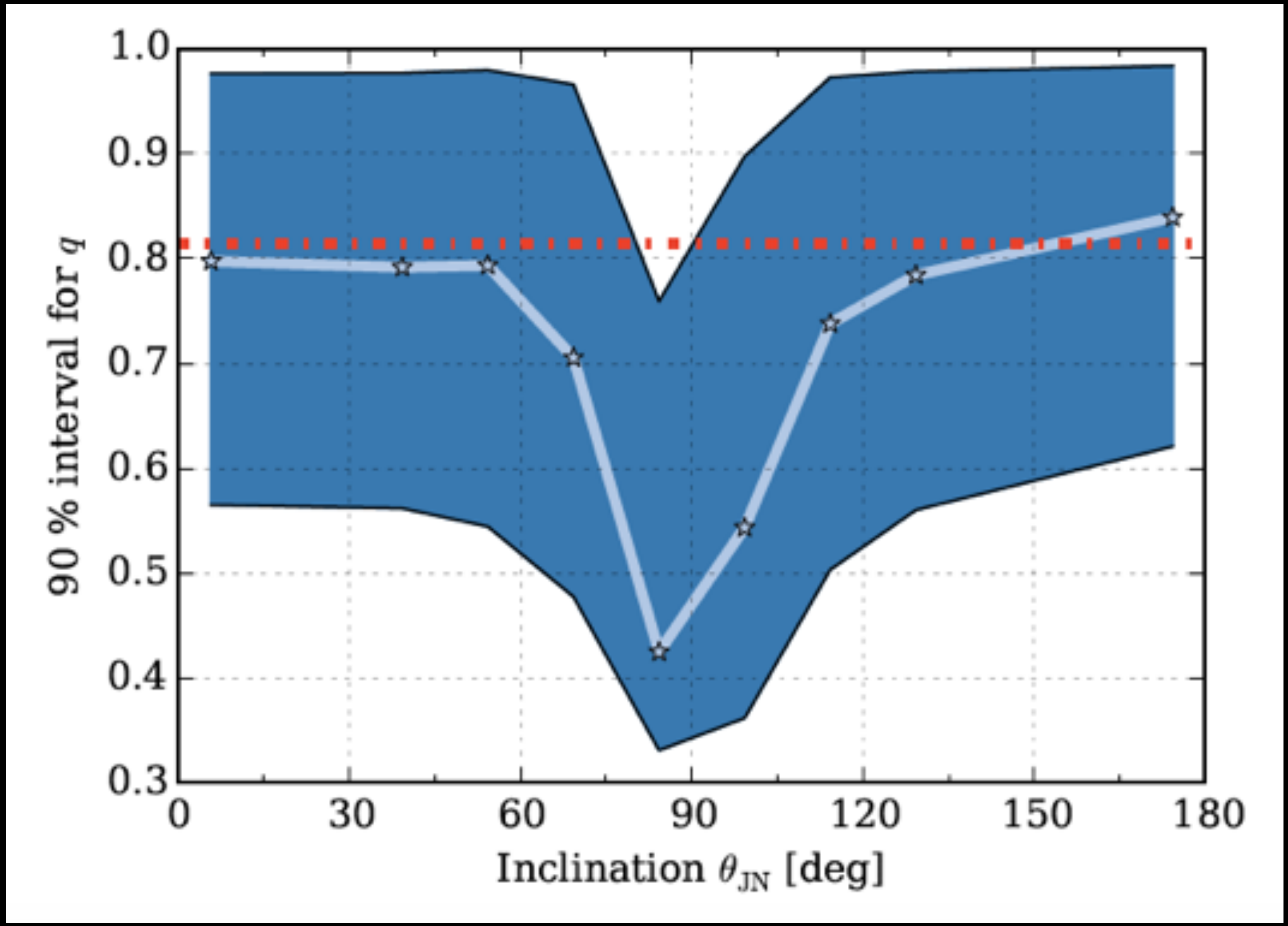
NR simulations around GW150914

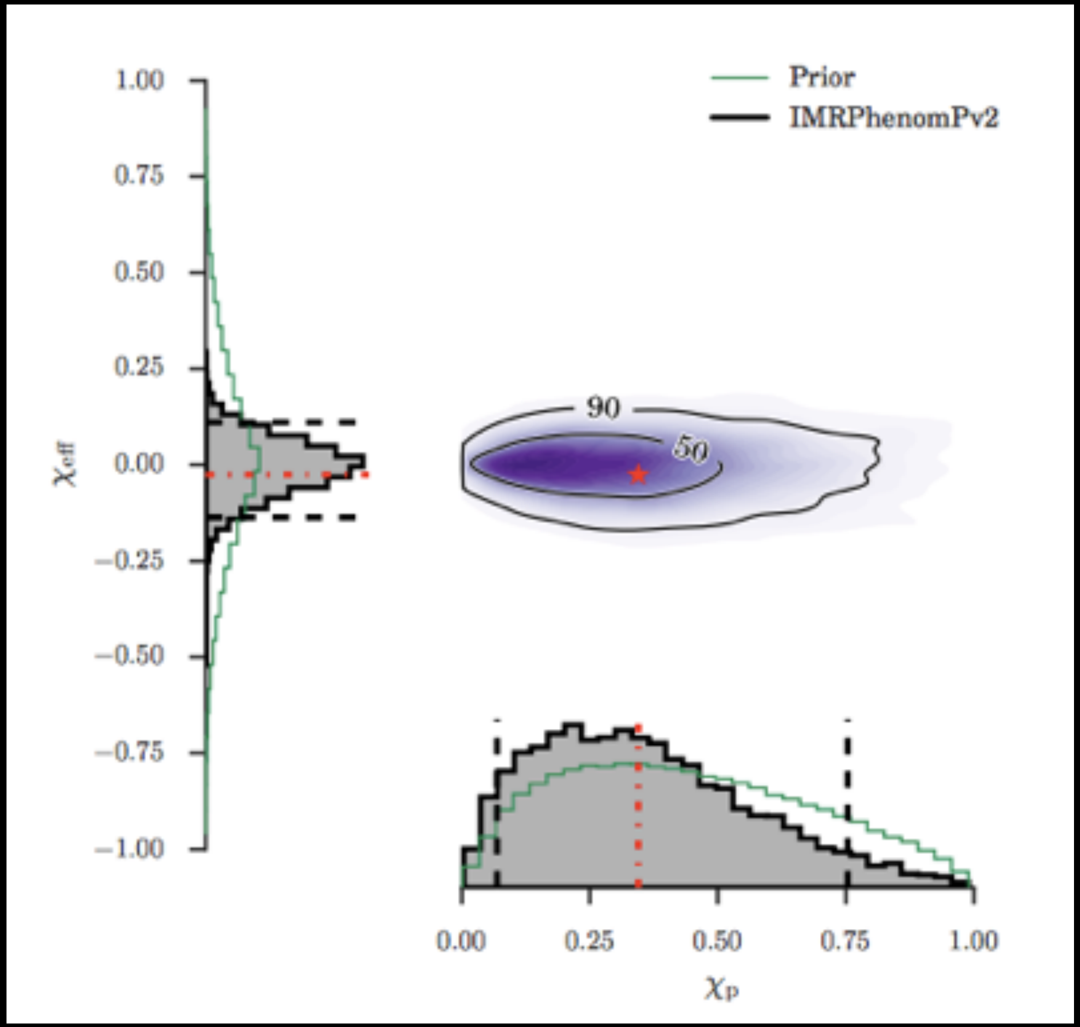




Inspiral
Merger
Ring-down
1.0
0.5
0.0
-0.5
-1.0
Strain (10^{-21})
Numerical relativity
Reconstructed (template)

Waveform model systematics





[Preliminary]

Local modelling

- Several hundred NR simulations performed
- Cross-check of parameter estimates
- Reduced-order-quadrature model
- PhenomP tuned through merger

Future observations

- SNR 25 at the accuracy limit of current models
- GW150914 was in the best-modelled region
- Better models need
  - Higher harmonics
  - Precession physics through merger
  - More accuracy (!)

- BUT: degeneracies will not evaporate!