

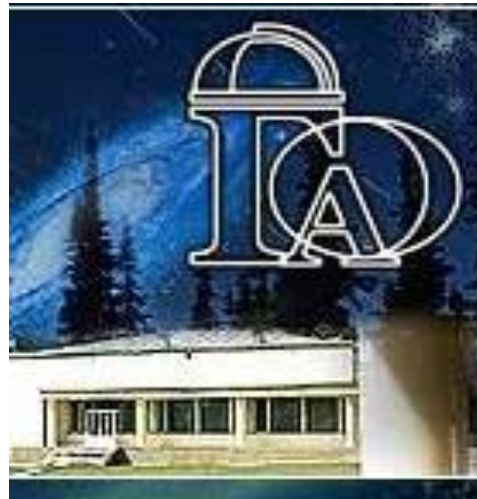
The DRAGON globular cluster simulations: a million stars, black holes and gravitational waves.

Peter Berczik

ARI, Heidelberg University, Germany

NAOC, Chinese Academy of Sciences, Beijing

MAO, National Academy of Sciences of Ukraine, Kiev



2016 May 24, AEI, Hannover, Germany.

Items:

-Collaborators + Grants...

**-GPU hardware / software. GPU clusters
(China, Germany, Poland, UK, Ukraine)...**

**-Astrophysical results of BBH evolution in
galactic GC's...**

Collaborators:

-<http://silkroad.bao.ac.cn/web/>

-Rainer Spurzem (NAOC, Beijing; ARI, Heidelberg Univ.)

-Long Wang (!!!) & Thijs Kouwenhoven (KIAA, Beijing)

-Sverre Aarseth (IA, Cambridge)

-Mirek Giersz & Abbas Askar (Copernicus AC, Warsaw)

-Thorsten Naab & Riko Schadow (MPIA, Garching)

“Silk Road Project” – CAS, China

SFB 881 “The Milky Way System” – DFG, Germany

GPU cluster “laohu” - ZDYZ2008-2, NAOC, CAS

GPU cluster “kepler” - I/80 041-043 and I/81 396, VW

GPU cluster “golowood” – GRID/GPU, MAO, NASU

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GPU Hardware

<http://gpgpu.org>

<http://www.nvidia.com>



2007: GeForce 8800 GTX, 128 SP, 768 MB

2008: GeForce 9800 GTX+, 128 SP, 512 MB

2009: GeForce GTX 280, 240 SP, 1 GB

2010: GeForce GTX 480, 480 SP, 1.5 GB

2011: GeForce GTX 580, 512 SP, 1.5 GB

2012: GeForce GTX 680, 1536 SP, 2 GB

2013: GeForce GTX TITAN, 2688 SP, 6 GB

2015: GeForce GTX TITAN X, 3072 SP, 12 GB

Milky Way GPU cluster.

SFB 881 – The Milky Way System

Collaboration with FZ Jülich, Germany

206 nodes x 24 = 4944 CPU cores (@ 2.8 GHz)

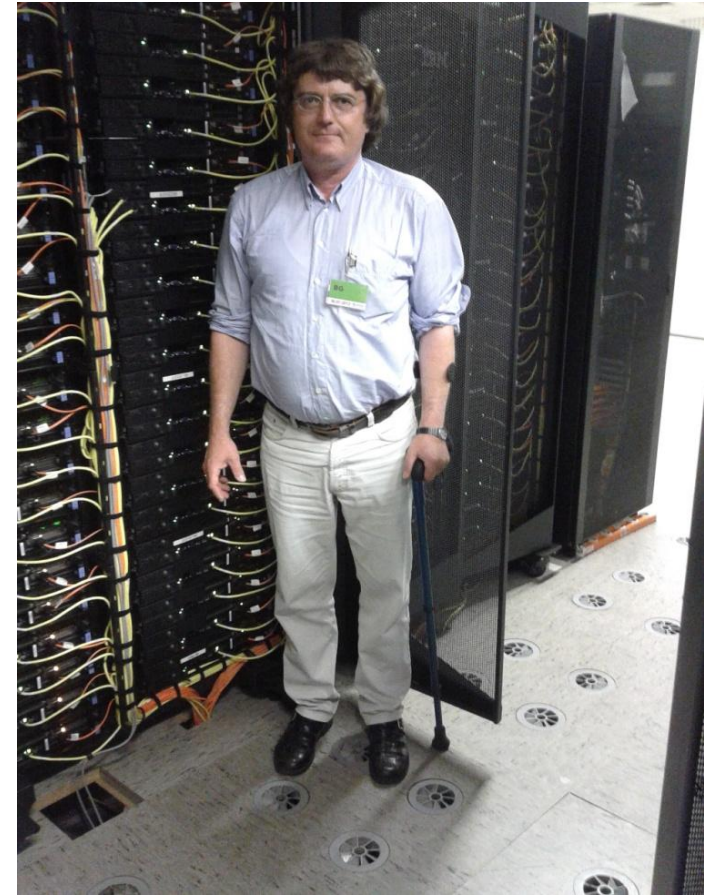
206 x 96 GB ~ 20 TB RAM CPU memory

408 GPUs M2070/M2050 ~ 200k GPU threads

~ 2 TB GPU device memory

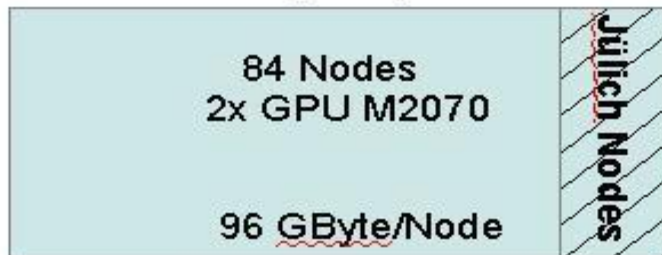
since mid. 2012 jointly operated.

nodes "judge123 - judge206" – MW part.

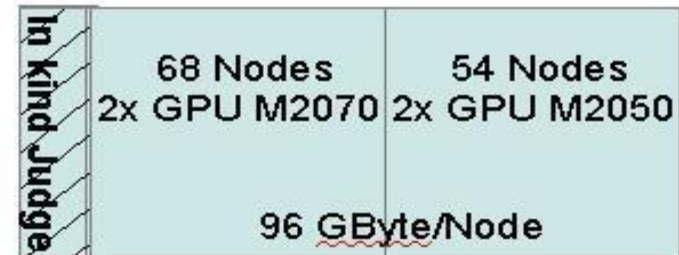


GPU Cluster

Milky Way



Judge



Kepler GPU cluster

Kepler GPU cluster

12 nodes = 12 x 16 = 192 CPU cores (@ 2 GHz)

12 x 64 GB = 768 GB RAM CPU memory

12 GPUs K20m = 12 x 2496 ~ 30k GPU threads

12 x 4.8 GB ~ 57 GB GPU device memory

4 x Xilinx Virtex-6 FPGA (ML 605)

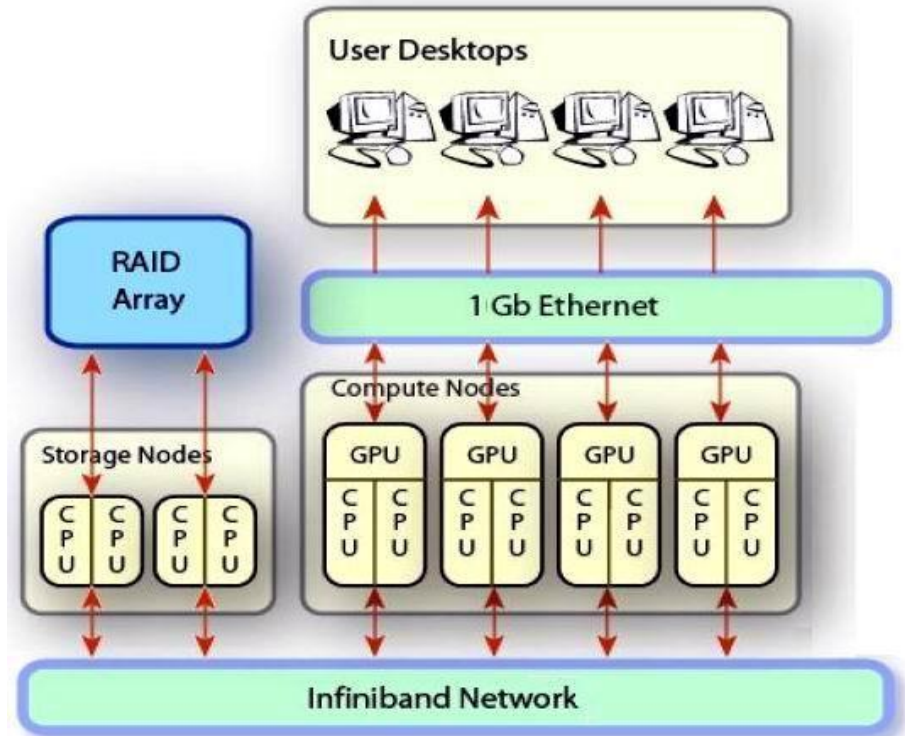
since beg. 2013 operated.



NAOC, China 85 node K20 cluster



MAO, Kiev 16 node GPU cluster

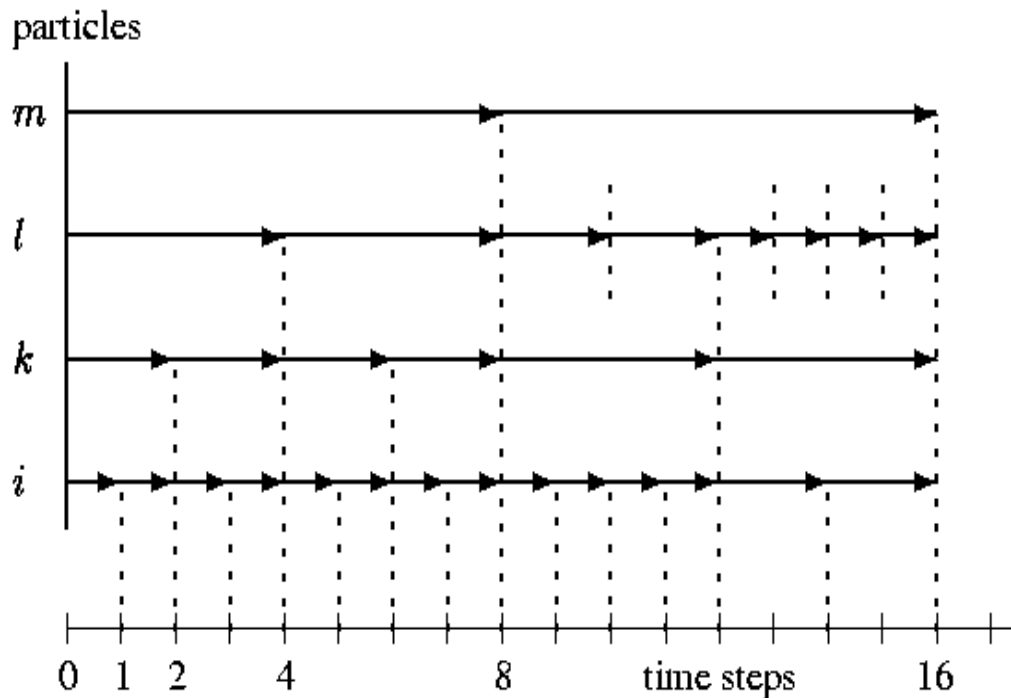
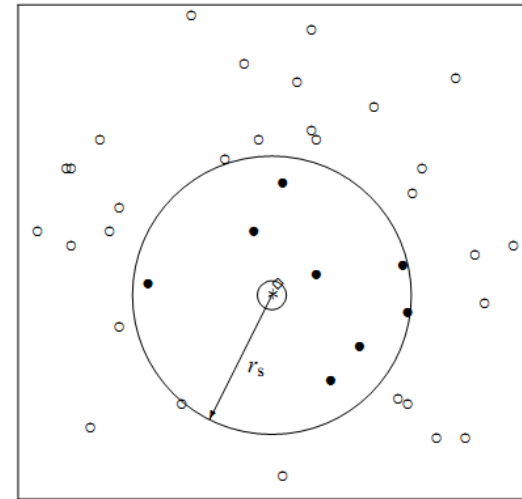


- 9 x 2 quad-core Xeon 2.33 GHz
- 16 x GF GTX 660
- Speed: ~12 Tflops
- RAID-5: ~5 TB
- Infiniband Network: ~20 Gb/s
- Cost: ~200k UAH
- Funding: NASU GRID

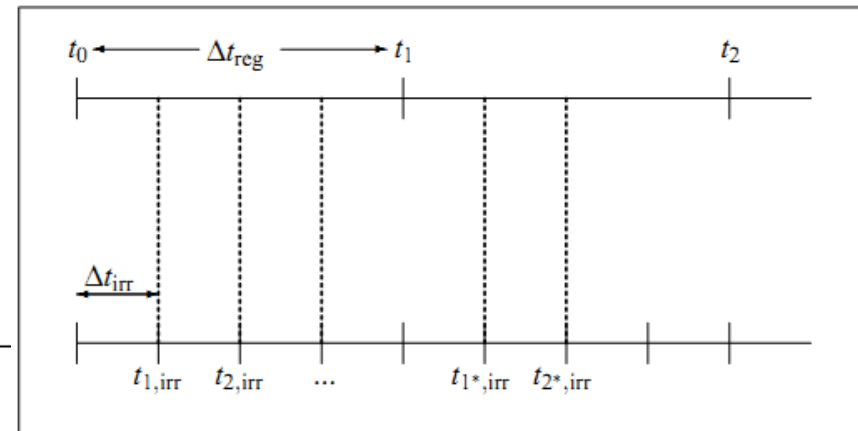
Our CPU/GPU N-body (AC) code

Hierarchical Individual Block Time Steps Ahmad – Cohen neighbor scheme

$$\frac{d^2 \vec{r}_i}{dt^2} = \vec{a}_i \quad \Delta t = \sqrt{\eta \frac{|\vec{a}| |\vec{a}^{(2)}| + |\vec{a}|^2}{|\vec{a}| |\vec{a}^{(3)}| + |\vec{a}^{(2)}|^2}}$$

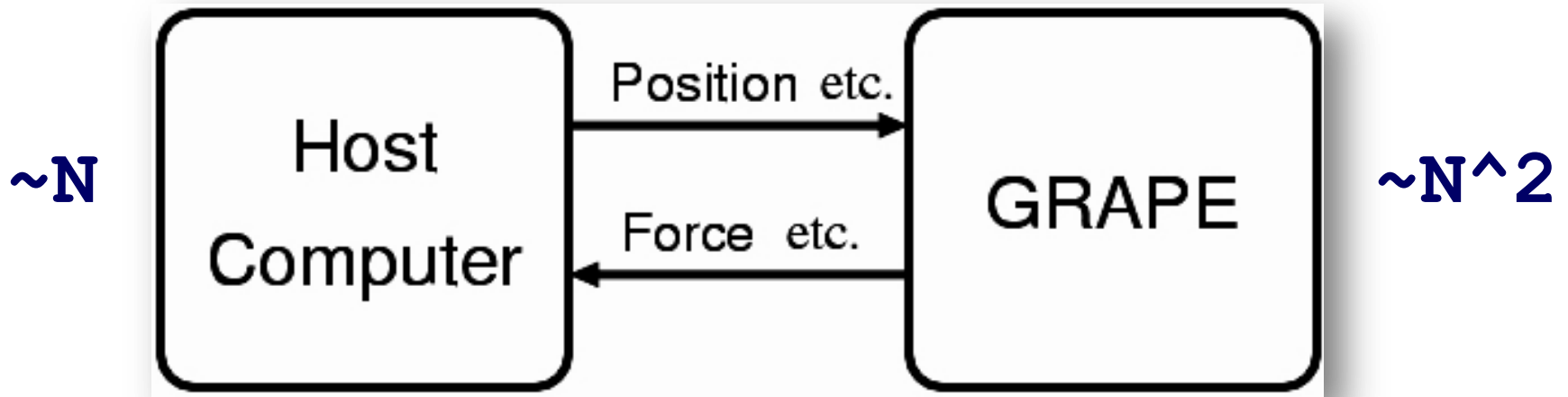


4th order Hermite scheme



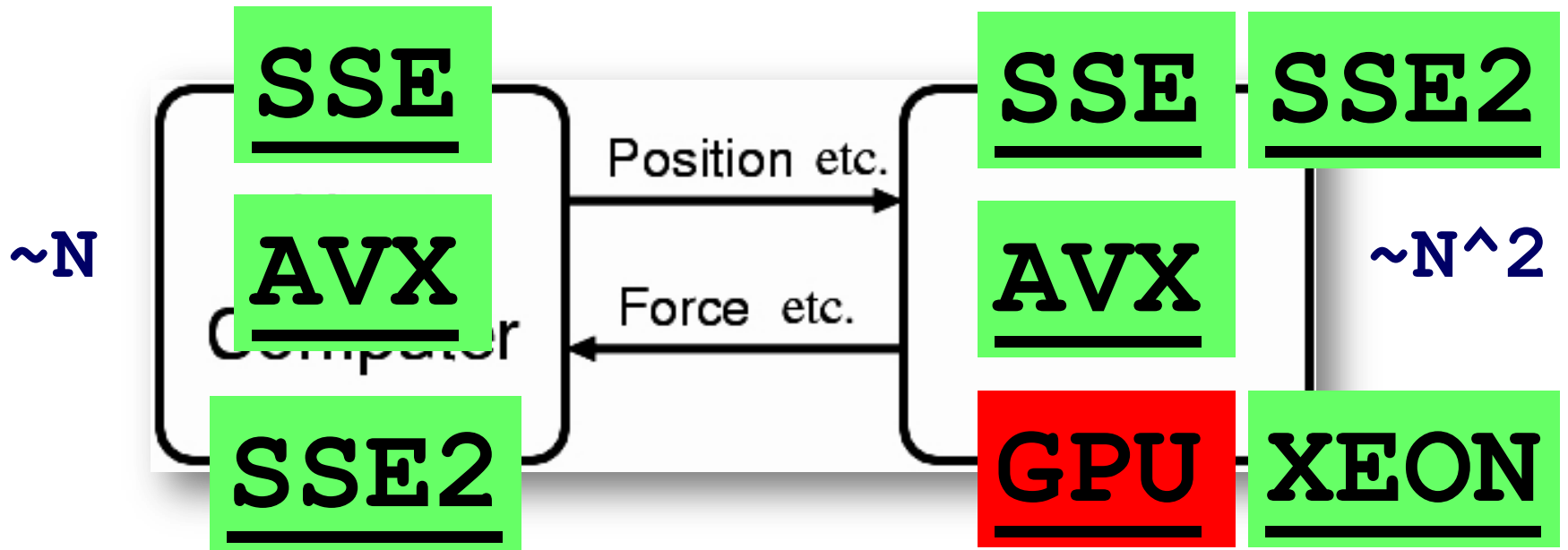
<https://github.com/lwang-astro/betanb6pp>

Our CPU/GPU N-body (AC) code



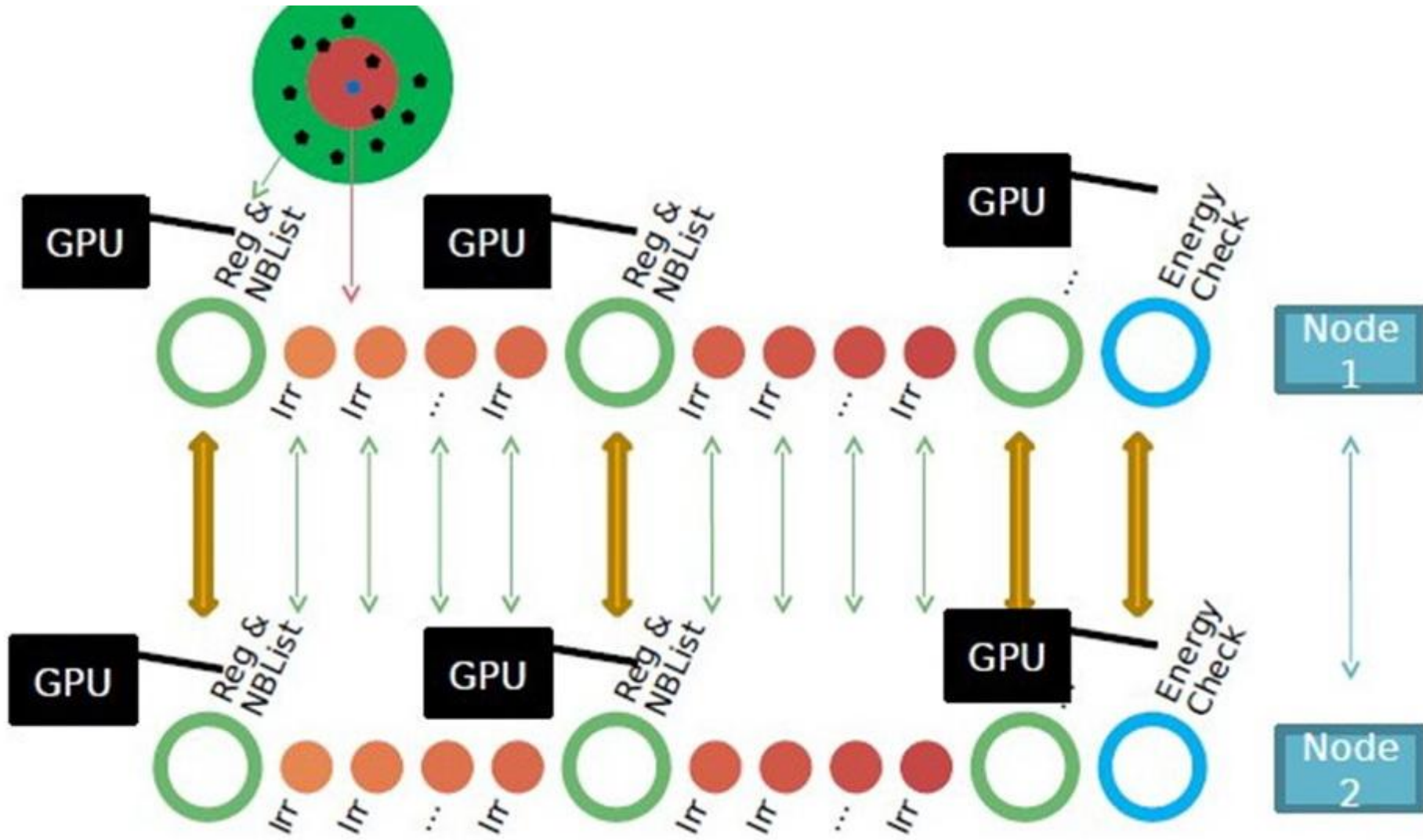
$$\vec{a}_i = \sum_{j=1; j \neq i}^N \vec{f}_{ij} \quad \vec{f}_{ij} = - \frac{G \cdot m_j}{(r_{ij}^2 + \varepsilon^2)^{3/2}} \vec{r}_{ij}$$

Our CPU/GPU N-body (AC) code



$$\vec{a}_i = \sum_{j=1; j \neq i}^N \vec{f}_{ij} \quad \vec{f}_{ij} = - \frac{G \cdot m_j}{(r_{ij}^2 + \varepsilon^2)^{3/2}} \vec{r}_{ij}$$

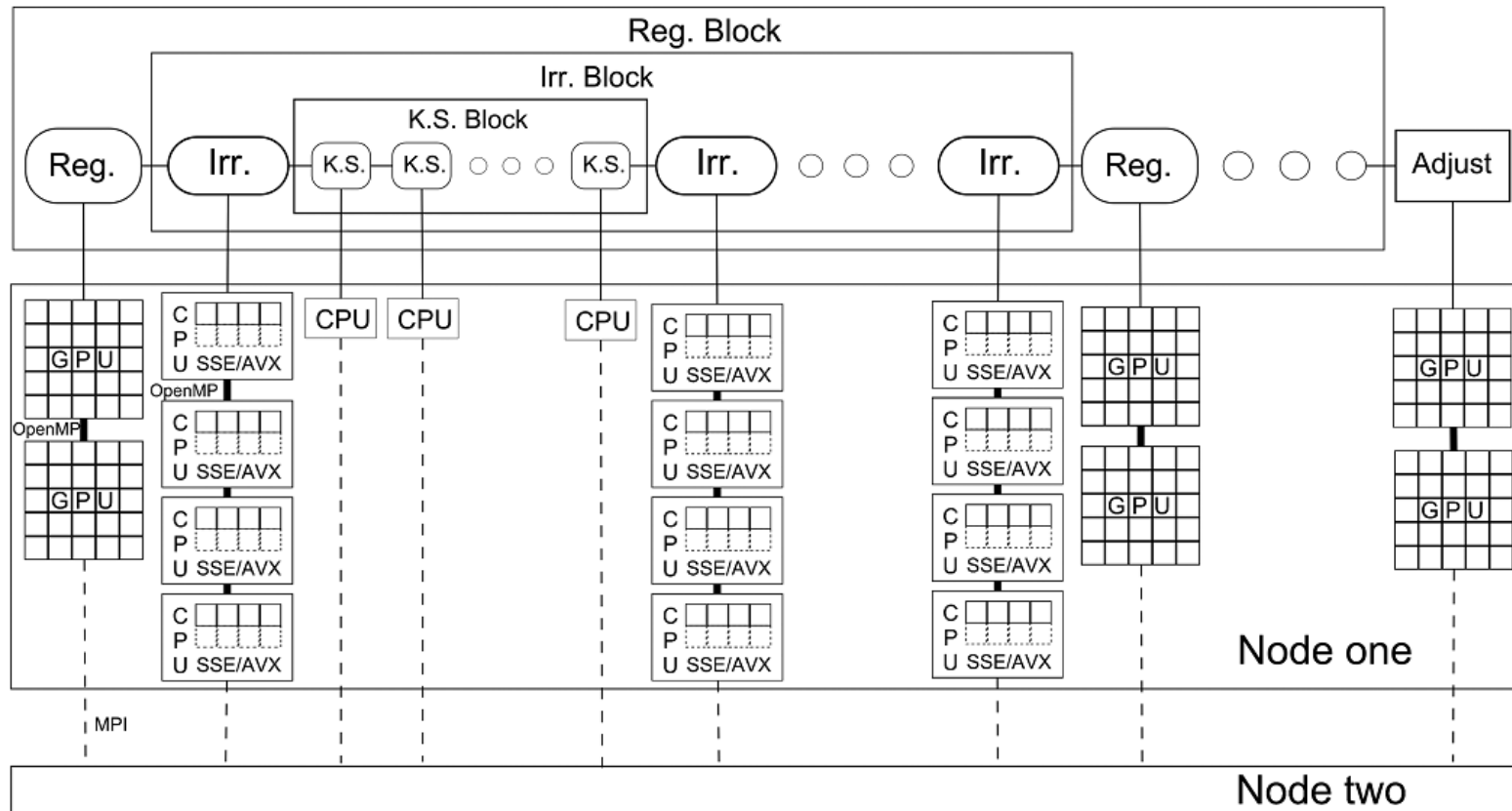
Our CPU/GPU N-body (AC) code



<https://github.com/lwang-astro/betanb6pp>

NBODY6++GPU: ready for the gravitational million-body problem

Long Wang,^{1,2}★ Rainer Spurzem,^{3,4,5,1} Sverre Aarseth,⁶ Keigo Nitadori,⁷
 Peter Berczik,^{3,4,5,8} M. B. N. Kouwenhoven^{1,2} and Thorsten Naab⁹



CPU/GPU **N-body6++**

Key Question 1. When will we see the first star-by-star N -body model of a globular cluster?

- Honest N -body simulation
- Reasonable mass at 12 Gyr ($\sim 5 \times 10^4 M_{\odot}$)
- Reasonable tide (circular galactic orbit will do)
- Reasonable IMF (e.g. Kroupa)
- Reasonable binary fraction (a few percent)
- Any initial model you like (Plummer will do)
- A submitted paper (astro-ph will do)

An inducement: a bottle of single malt Scotch whisky worth €50

The million-body problem at last!



The bottle of whisky is awarded to
Long Wang (Beijing)

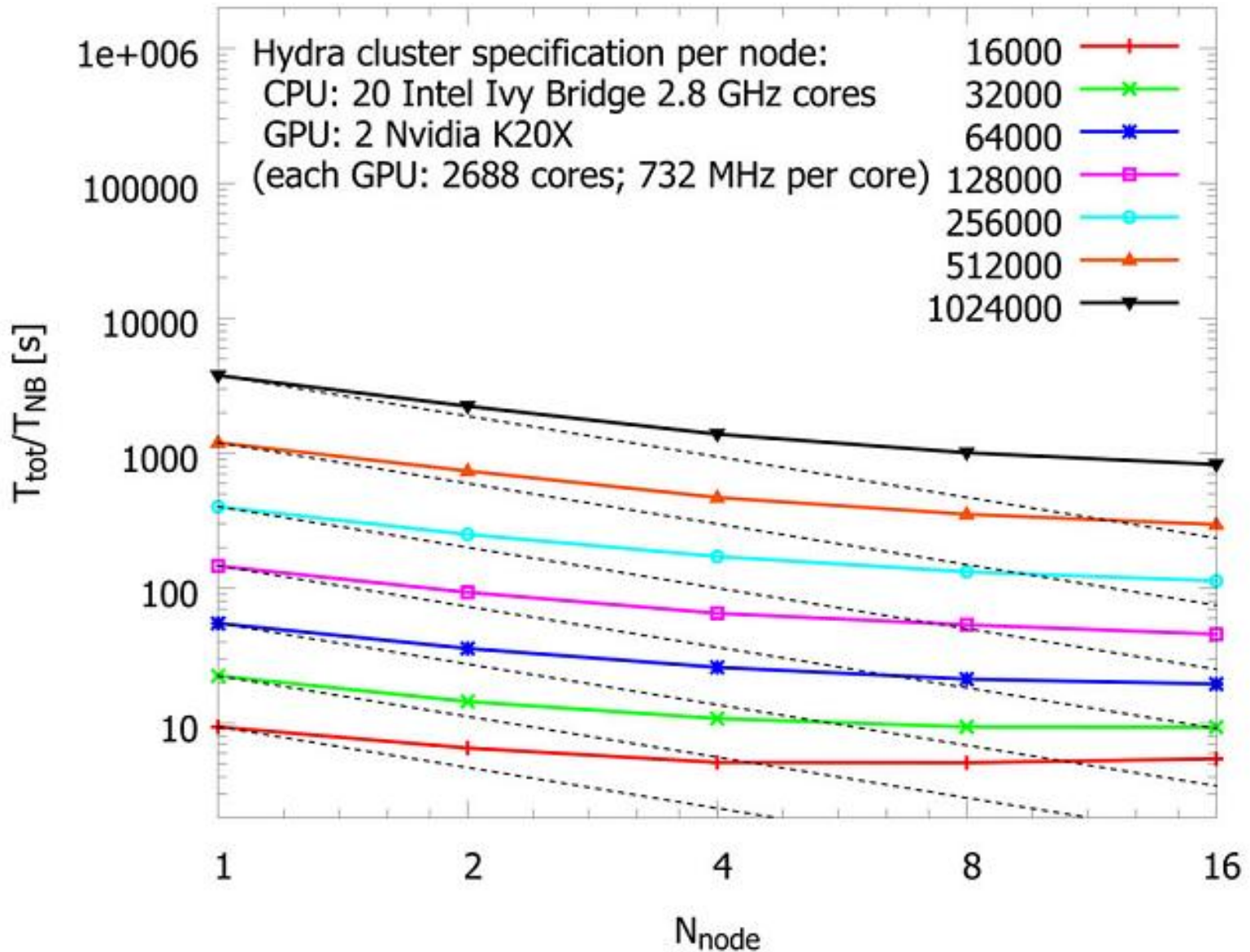


CPU/GPU N-body6++



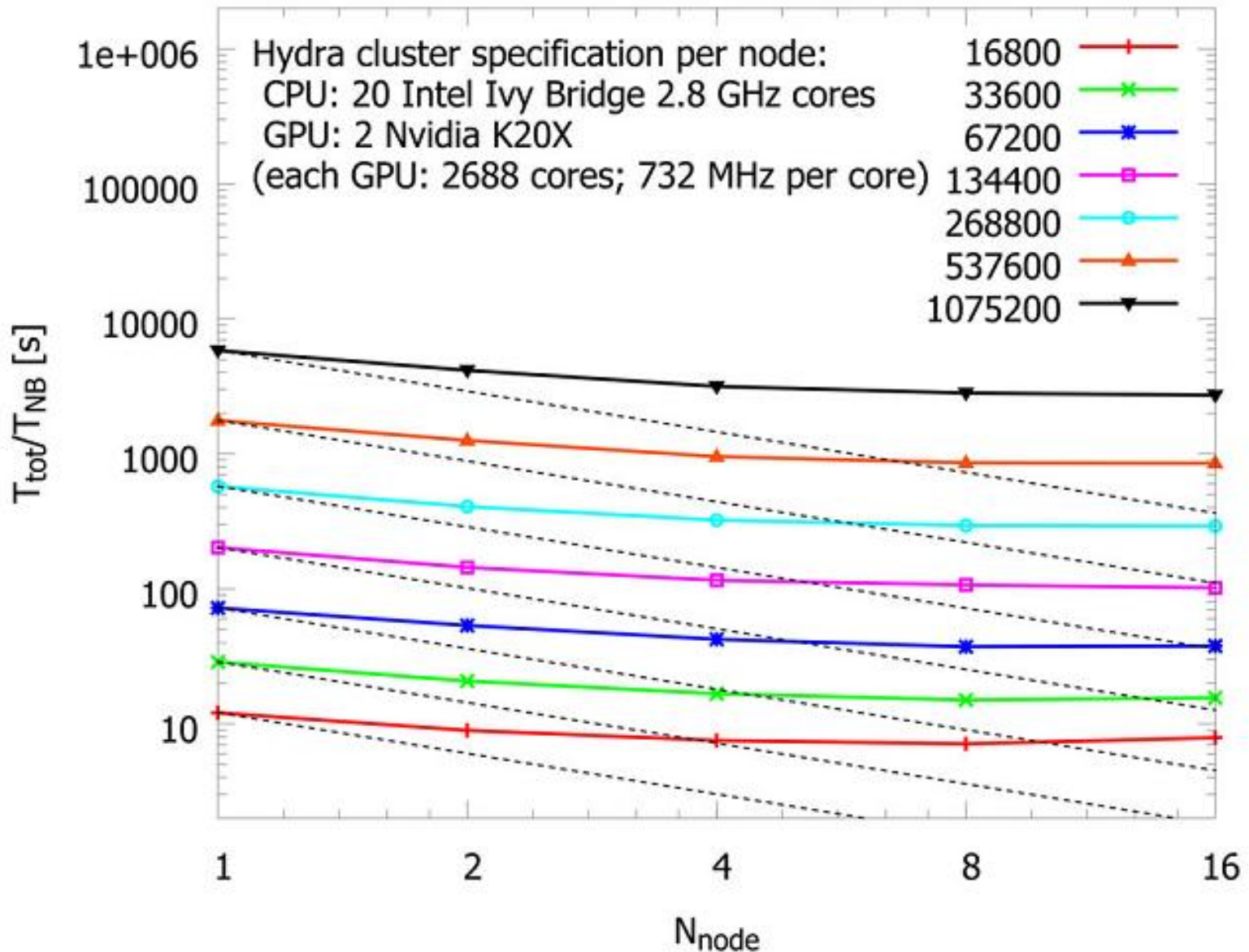
CPU/GPU N-body6++

0% bin



CPU/GPU N-body6++

5% bin



Performance analysis of parallel gravitational N -body codes on large GPU clusters

Si-Yi Huang¹, Rainer Spurzem^{1,2,4} and Peter Berczik^{1,3,4}

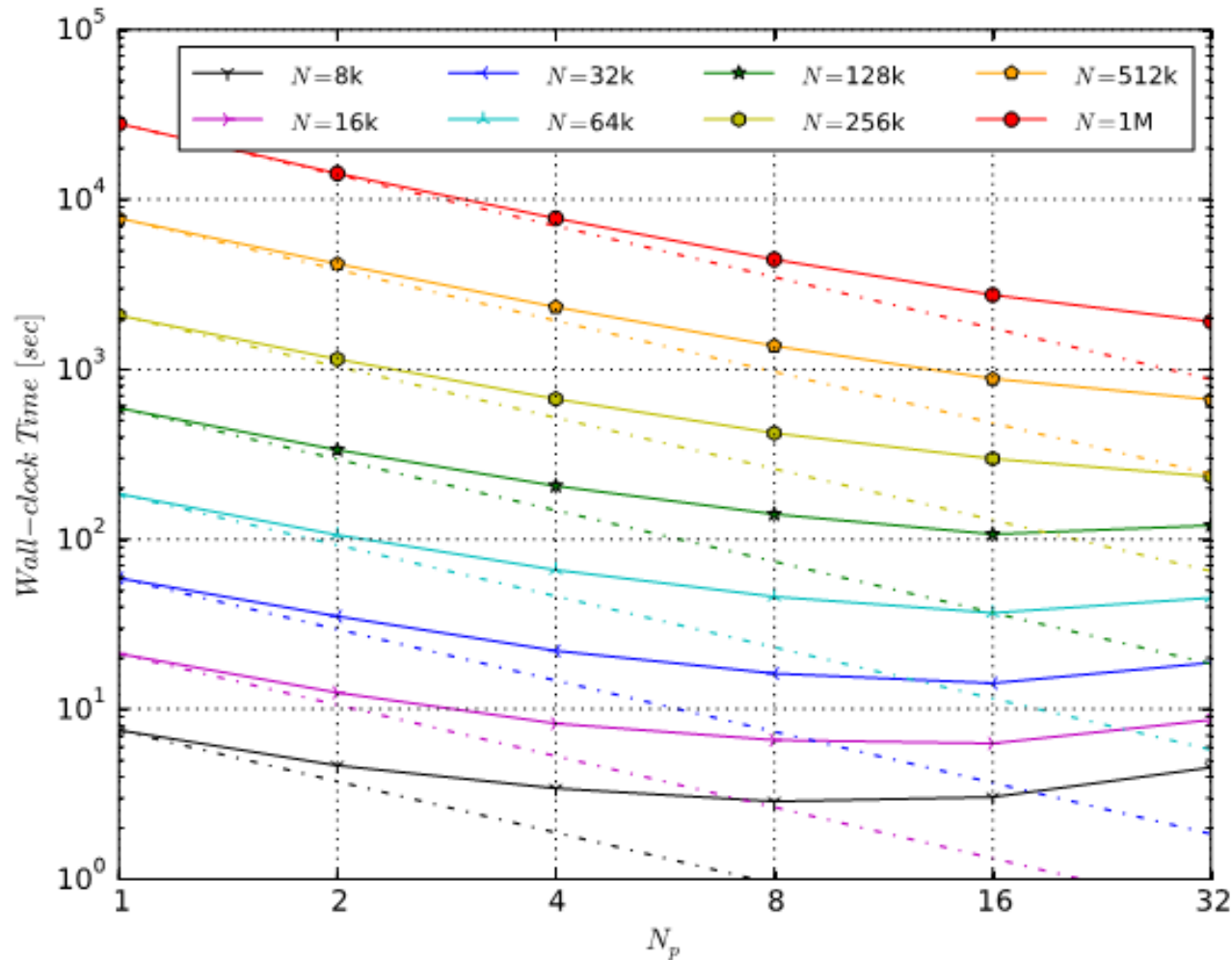


Table 2 Main Components of NBODY6++

Description	Timing variable	Expected scaling		Fitting value [s]
		N	N_p	
Regular force computation	T_{reg}	$\mathcal{O}(N_{\text{reg}} \times N)$	$\mathcal{O}(N_p^{-1})$	$(2.2 \times 10^{-9} \times N^{2.11} + 10.43) \times N_p^{-1}$
Irregular force computation	T_{irr}	$\mathcal{O}(N_{\text{irr}} \times \langle N_{nb} \rangle)$	$\mathcal{O}(N_p^{-1})$	$(3.9 \times 10^{-7} \times N^{1.76} - 16.47) \times N_p^{-1}$
Prediction	T_{pre}	$\mathcal{O}(N^{kn_p})$	$\mathcal{O}(N_p^{-kp_p})$	$(1.2 \times 10^{-6} \times N^{1.51} - 3.58) \times N_p^{-0.5}$
Data moving	T_{mov}	$\mathcal{O}(N^{kn_{m1}})$	$\mathcal{O}(1)$	$2.5 \times 10^{-6} \times N^{1.29} - 0.28$
MPI communication (Reg.)	T_{mcr}	$\mathcal{O}(N^{kn_{cr}})$	$\mathcal{O}(kp_{cr} \times \frac{N_p-1}{N_p})$	$(3.3 \times 10^{-6} \times N^{1.18} + 0.12)(1.5 \times \frac{N_p-1}{N_p})$
MPI communication (Irr.)	T_{mci}	$\mathcal{O}(N^{kn_{ci}})$	$\mathcal{O}(kp_{ci} \times \frac{N_p-1}{N_p})$	$(3.6 \times 10^{-7} \times N^{1.40} + 0.56)(1.5 \times \frac{N_p-1}{N_p})$
Synchronization	T_{syn}	$\mathcal{O}(N^{kn_s})$	$\mathcal{O}(N_p^{kp_s})$	$(4.1 \times 10^{-8} \times N^{1.34} + 0.07) \times N_p$
Sequential parts on host	T_{host}	$\mathcal{O}(N^{kn_h})$	$\mathcal{O}(1)$	$4.4 \times 10^{-7} \times N^{1.49} + 1.23$

PN routine

$$\mathbf{a}_{\text{NoSpin}} = \mathbf{a}_{\text{N}} + \frac{1}{c^2} \mathbf{a}_{1\mathcal{PN}} + \frac{1}{c^4} \mathbf{a}_{2\mathcal{PN}} + \frac{1}{c^5} \mathbf{a}_{2.5\mathcal{PN}} + \frac{1}{c^6} \mathbf{a}_{3\mathcal{PN}} + \frac{1}{c^7} \mathbf{a}_{3.5\mathcal{PN}} + \mathcal{O}\left(\frac{1}{c^8}\right),$$

Blanchet 2006

Faye et al. 2006

Tagoshi et al. 2001

$$\mathbf{a}_{\text{Spin}} = \mathbf{a}_{\text{NoSpin}} + \frac{1}{c^3} \mathbf{a}_{1.5\mathcal{PN},\text{SO}} + \frac{1}{c^4} \mathbf{a}_{2\mathcal{PN},\text{SS}} + \frac{1}{c^5} \mathbf{a}_{2.5\mathcal{PN},\text{SO}},$$

$$S^{\text{true}} = \chi \frac{Gm^2}{c}$$

$$\chi \in [0, 1]$$

$$\frac{d\mathbf{v}}{dt} = -\frac{Gm}{r^2} [(1 + A)\mathbf{n} + B\mathbf{v}].$$

Kupi et al. 2006; Berentzen et al. 2009; Brem 2011;
Sobolenko et al. 2016

$$\frac{d\vec{v}}{dt} = -\frac{Gm}{r^2} [(1 + A)\vec{n} + B\vec{v}].$$

$$\begin{aligned}
A = & \frac{1}{c^2} \left\{ -\frac{3\dot{r}^2\eta}{2} + v^2 + 3\eta v^2 - \frac{Gm}{r}(4 + 2\eta) \right\} \\
& + \frac{1}{c^4} \left\{ \frac{15\dot{r}^4\eta}{8} - \frac{45\dot{r}^4\eta^2}{8} - \frac{9\dot{r}^2\eta v^2}{2} + 6\dot{r}^2\eta^2 v^2 + 3\eta v^4 - 4\eta^2 v^4 \right. \\
& \left. + \frac{Gm}{r}(-2\dot{r}^2 - 25\dot{r}^2\eta - 2\dot{r}^2\eta^2 - \frac{13\eta v^2}{2} + 2\eta^2 v^2) + \frac{G^2 m^2}{r^2}(9 + \frac{87\eta}{4}) \right\} \\
& + \frac{1}{c^5} \left\{ -\frac{24\dot{r}\eta v^2}{5} \frac{Gm}{r} - \frac{136\dot{r}\eta}{15} \frac{G^2 m^2}{r^2} \right\} \\
& + \frac{1}{c^6} \left\{ -\frac{35\dot{r}^6\eta}{16} + \frac{175\dot{r}^6\eta^2}{16} - \frac{175\dot{r}^6\eta^3}{16} + \frac{15\dot{r}^4\eta v^2}{2} - \frac{135\dot{r}^4\eta^2 v^2}{4} + \frac{255\dot{r}^4\eta^3 v^2}{8} \right. \\
& - \frac{15\dot{r}^2\eta v^4}{2} + \frac{237\dot{r}^2\eta^2 v^4}{8} - \frac{45\dot{r}^2\eta^3 v^4}{2} + \frac{11\eta v^6}{4} - \frac{49\eta^2 v^6}{4} + 13\eta^3 v^6 \\
& + \frac{Gm}{r}(79\dot{r}^4\eta - \frac{69\dot{r}^4\eta^2}{2} - 30\dot{r}^4\eta^3 - 121\dot{r}^2\eta v^2 + 16\dot{r}^2\eta^2 v^2 + 20\dot{r}^2\eta^3 v^2 + \frac{75\eta v^4}{4} \\
& + 8\eta^2 v^4 - 10\eta^3 v^4) \\
& + \frac{G^2 m^2}{r^2}(\dot{r}^2 + \frac{22717\dot{r}^2\eta}{168} + \frac{11\dot{r}^2\eta^2}{8} - 7\dot{r}^2\eta^3 + \frac{615\dot{r}^2\eta\pi^2}{64} - \frac{20827\eta v^2}{840} + \eta^3 v^2 \\
& - \frac{123\eta\pi^2 v^2}{64}) \\
& \left. + \frac{G^3 m^3}{r^3}(-16 - \frac{1399\eta}{12} - \frac{71\eta^2}{2} + \frac{41\eta\pi^2}{16}) \right\} \\
& + \frac{1}{c^7} \left\{ \frac{Gm}{r}(\frac{366}{35}\eta v^4 + 12\eta^2 v^4 - 114v^2\eta\dot{r}^2 - 12\eta^2 v^2\dot{r}^2 + 112\eta\dot{r}^4) \right. \\
& + \frac{G^2 m^2}{r^2}(\frac{692}{35}\eta v^2 - \frac{724}{15}v^2\eta^2 + \frac{294}{5}\eta\dot{r}^2 + \frac{376}{5}\eta^2\dot{r}^2) \\
& \left. + \frac{G^3 m^3}{r^3}(\frac{3956}{35}\eta + \frac{184}{5}\eta^2) \right\}
\end{aligned}$$

$$\begin{aligned}
B = & \frac{1}{c^2} \{-4\dot{r} + 2\dot{r}\eta\} \\
& + \frac{1}{c^4} \left\{ \frac{9\dot{r}^3\eta}{2} + 3\dot{r}^3\eta^2 - \frac{15\dot{r}\eta v^2}{2} - 2\dot{r}\eta^2 v^2 + \frac{Gm}{r}(2\dot{r} + \frac{41\dot{r}\eta}{2} + 4\dot{r}\eta^2) \right\} \\
& + \frac{1}{c^5} \left\{ \frac{8\eta v^2}{5} \frac{Gm}{r} + \frac{24\eta}{5} \frac{G^2 m^2}{r^2} \right\} \\
& + \frac{1}{c^6} \left\{ -\frac{45\dot{r}^5\eta}{8} + 15\dot{r}^5\eta^2 + \frac{15\dot{r}^5\eta^3}{4} + 12\dot{r}^3\eta v^2 - \frac{111\dot{r}^3\eta^2 v^2}{4} - 12\dot{r}^3\eta^3 v^2 - \frac{65\dot{r}\eta v^4}{8} \right. \\
& + 19\dot{r}\eta^2 v^4 + 6\dot{r}\eta^3 v^4 \\
& + \frac{Gm}{r}(\frac{329\dot{r}^3\eta}{6} + \frac{59\dot{r}^3\eta^2}{2} + 18\dot{r}^3\eta^3 - 15\dot{r}\eta v^2 - 27\dot{r}\eta^2 v^2 - 10\dot{r}\eta^3 v^2) \\
& + \frac{G^2 m^2}{r^2}(-4\dot{r} - \frac{5849\dot{r}\eta}{840} + 25\dot{r}\eta^2 + 8\dot{r}\eta^3 - \frac{123\dot{r}\eta\pi^2}{32}) \left. \right\} \\
& + \frac{1}{c^7} \left\{ \frac{Gm}{r}(-\frac{626}{35}\eta v^4 - \frac{12}{5}\eta^2 v^4 + \frac{678}{5}\eta v^2\dot{r}^2 + \frac{12}{5}\eta^2 v^2\dot{r}^2 - 120\eta\dot{r}^4) \right. \\
& + \frac{G^2 m^2}{r^2}(\frac{164}{21}\eta v^2 + \frac{148}{5}\eta^2 v^2 - \frac{82}{3}\eta\dot{r}^2 - \frac{848}{15}\eta^2\dot{r}^2) \\
& \left. + \frac{G^3 m^3}{r^3}(-\frac{1060}{21}\eta - \frac{104}{5}\eta^2) \right\}
\end{aligned}$$

L. Blanchet. Gravitational Radiation from Post-Newtonian Sources and Inspiralling Compact Binaries. *Living Reviews in Relativity*, 9:4+, June 2006.

$$\frac{d\vec{v}}{dt} = \vec{B}_{\text{NoSpin}} + \frac{1}{c^2}\vec{B}_{1.5PN\text{SO}} + \frac{1}{c^4}\vec{B}_{2PN\text{SS}} + \frac{1}{c^4}\vec{B}_{2.5PN\text{SO}}$$

$$h^{ij} = \frac{2G\mu}{Dc^4} \left[Q^{ij} + P^{0.5} Q^{ij} + PQ^{ij} + PQ_{\text{SO}}^{ij} + P^{1.5} Q^{ij} + \dots \right]_{\text{TT}},$$

$$Q^{ij} = 2 \left[v^i v^j - \frac{Gm}{r} n^i n^j \right] \quad h^{ij} \approx \frac{4G\mu}{Dc^4} \left[v^i v^j - \frac{Gm}{r} n^i n^j \right]$$

$$\epsilon_+^{ij} = \hat{e}_y^i \hat{e}_y^j - \hat{e}_x^i \hat{e}_x^j$$

$$\epsilon_\times^{ij} = \hat{e}_x^i \hat{e}_y^j + \hat{e}_y^i \hat{e}_x^j,$$

$$\epsilon_+^{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\epsilon_\times^{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\hat{e}_x = (1, 0, 0)$$

$$\hat{e}_y = (0, 1, 0)$$

$$\hat{e}_z = (0, 0, 1),$$

$$h_+ = h^{ij} \epsilon_{ij}^+$$

$$h_\times = h^{ij} \epsilon_{ij}^\times.$$

Gravitational Radiation and the Motion of Two Point Masses

P. C. PETERS*†

California Institute of Technology, Pasadena, California

(Received 2 July 1964)

$$r = a(1 - e^2) / (1 + e \cos \psi)$$

$$a = -Gm_1m_2/2E,$$

$$L^2 = Gm_1^2m_2^2(m_1 + m_2)^{-1}a(1 - e^2).$$

$$\left\langle \frac{da}{dt} \right\rangle = -\frac{64 G^3 m_1 m_2 (m_1 + m_2)}{5 c^5 a^3 (1 - e^2)^{7/2}} \left(1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 \right)$$

$$\left\langle \frac{de}{dt} \right\rangle = -\frac{304 G^3 m_1 m_2 (m_1 + m_2)}{15 c^5 a^4 (1 - e^2)^{5/2}} \left(1 + \frac{121}{304} e^2 \right)$$

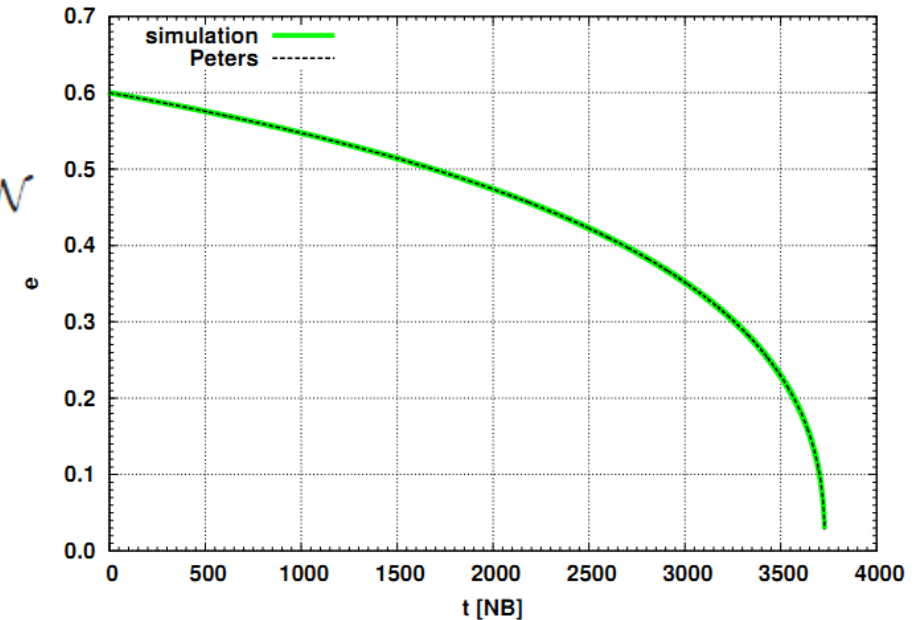
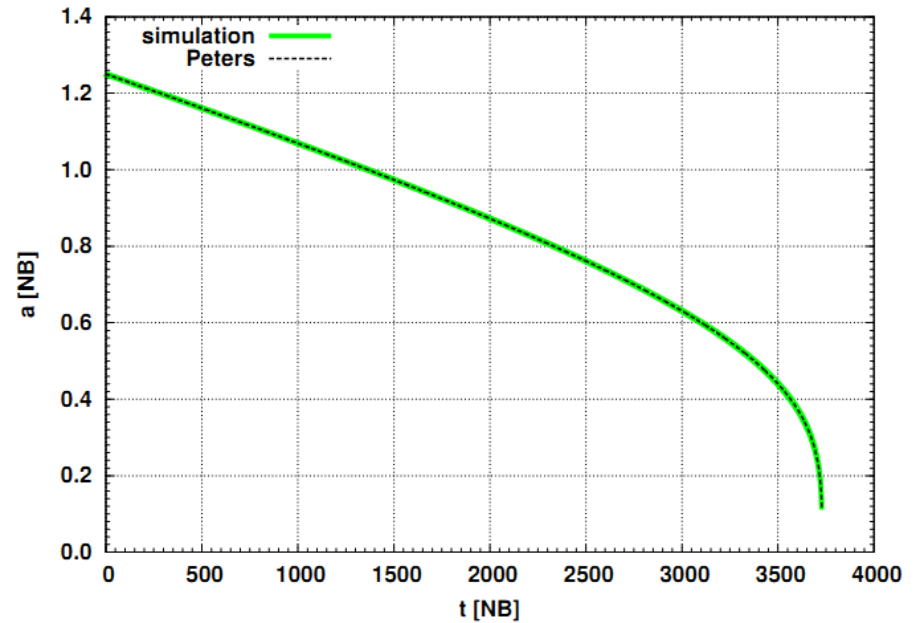
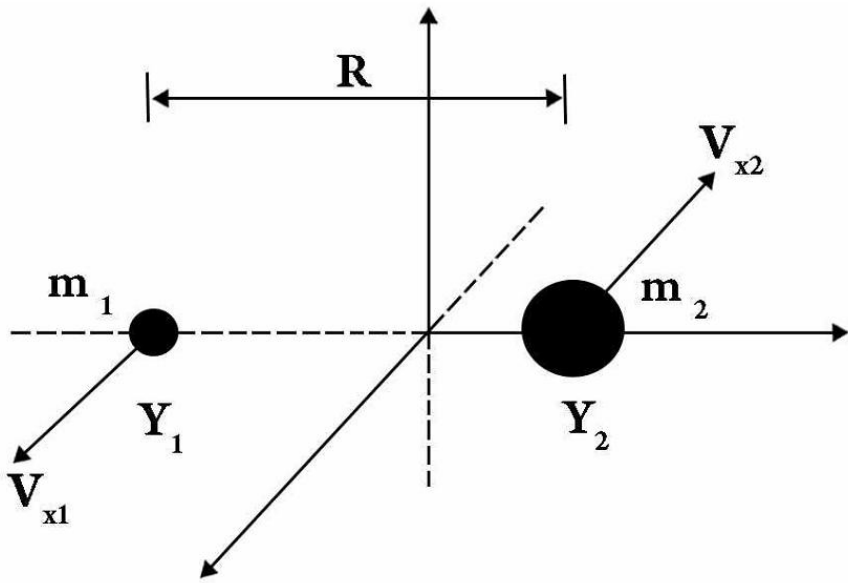
$$T \sim a / (da/dt)$$

$$\beta = \frac{64 G^3 m_1 m_2 (m_1 + m_2)}{5 c^5}$$

$$T_c(a_0) = a_0^4 / (4\beta)$$

$$T(a_0, e_0) \approx (768/425) T_c(a_0) (1 - e_0^2)^{7/2}$$

PN routine



$$M_{\bullet\text{tot}} = 2 \text{ [NB]}, q = 1, R = 1 \text{ [NB]},$$

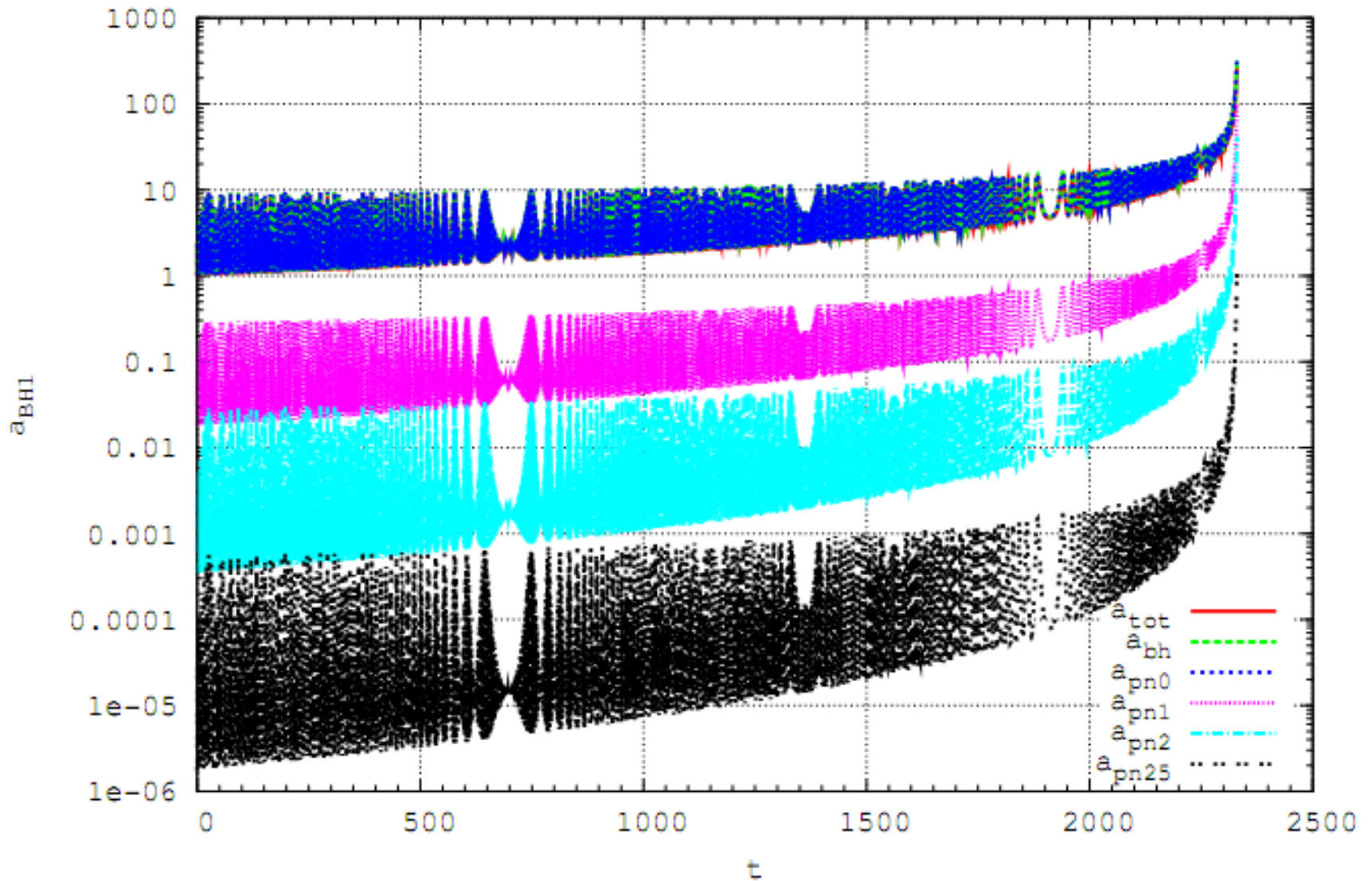
$$e_0 = 0.6, c = 15 \text{ [NB]} \text{ with just turning on } 2.5PN$$

$$\left\langle \frac{da}{dt} \right\rangle = -\frac{64 G^3 m_1 m_2 (m_1 + m_2)}{5 c^5 a^3 (1 - e^2)^{7/2}} \left(1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 \right)$$

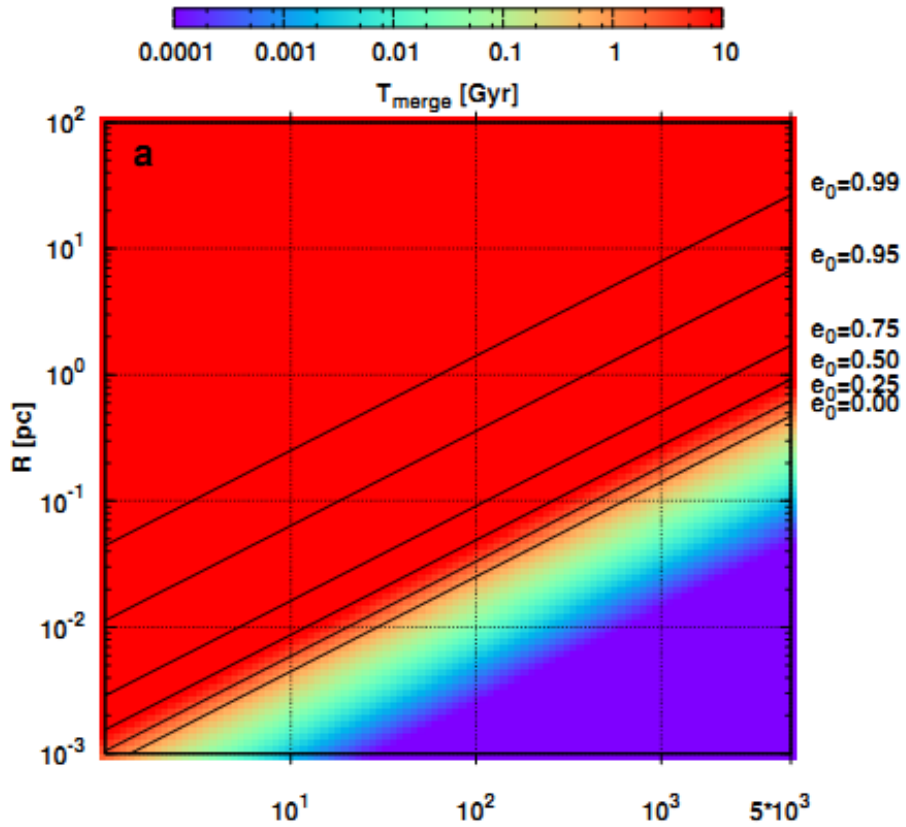
$$\left\langle \frac{de}{dt} \right\rangle = -\frac{304 G^3 m_1 m_2 (m_1 + m_2)}{15 c^5 a^4 (1 - e^2)^{5/2}} \left(1 + \frac{121}{304} e^2 \right)$$

PN routine

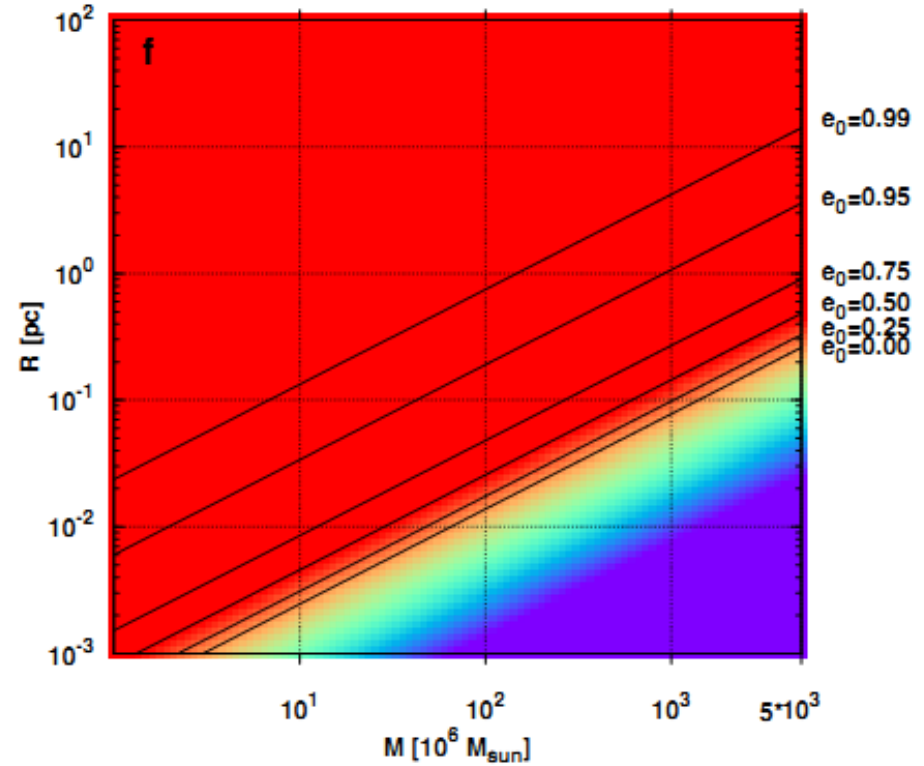
TWO BODY: $G=1$, $c=20$, $m_{\text{BH1}}=m_{\text{BH2}}=1.0$, $R(0) = +/-0.5$, $V(0) = +/-0.5$



PN routine



$q=1.00$



$q=0.02$

[Sobolenko et al. 2016](#)

Items:

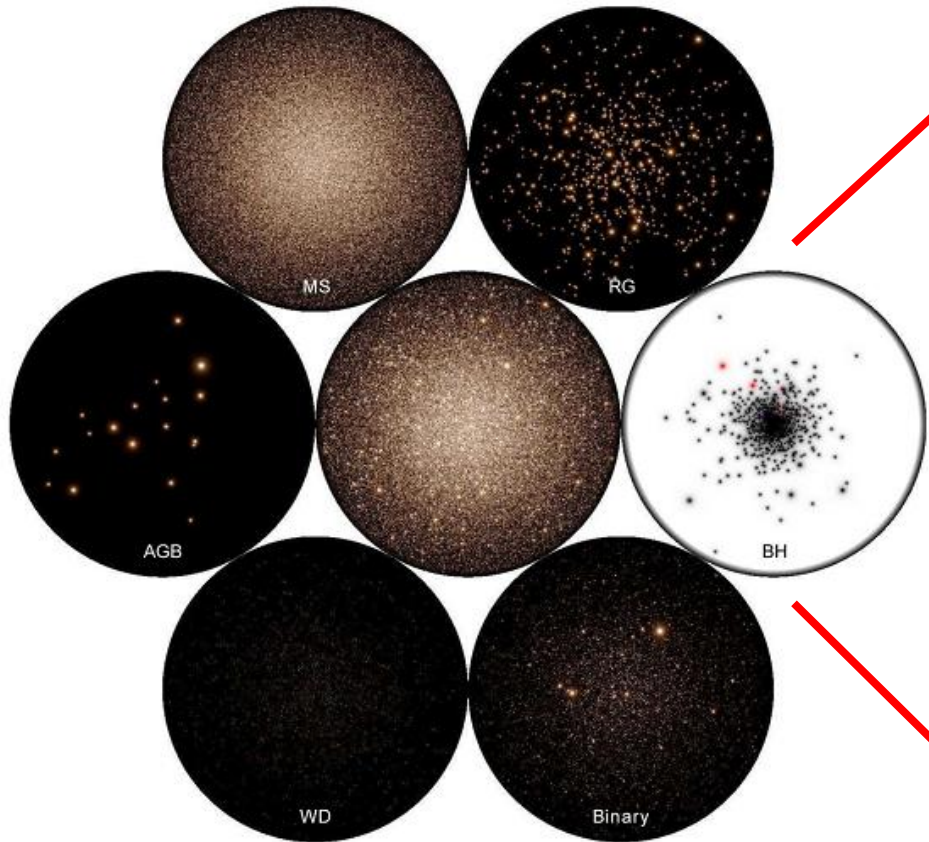
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(China, Germany, Poland, UK, Ukraine)...**

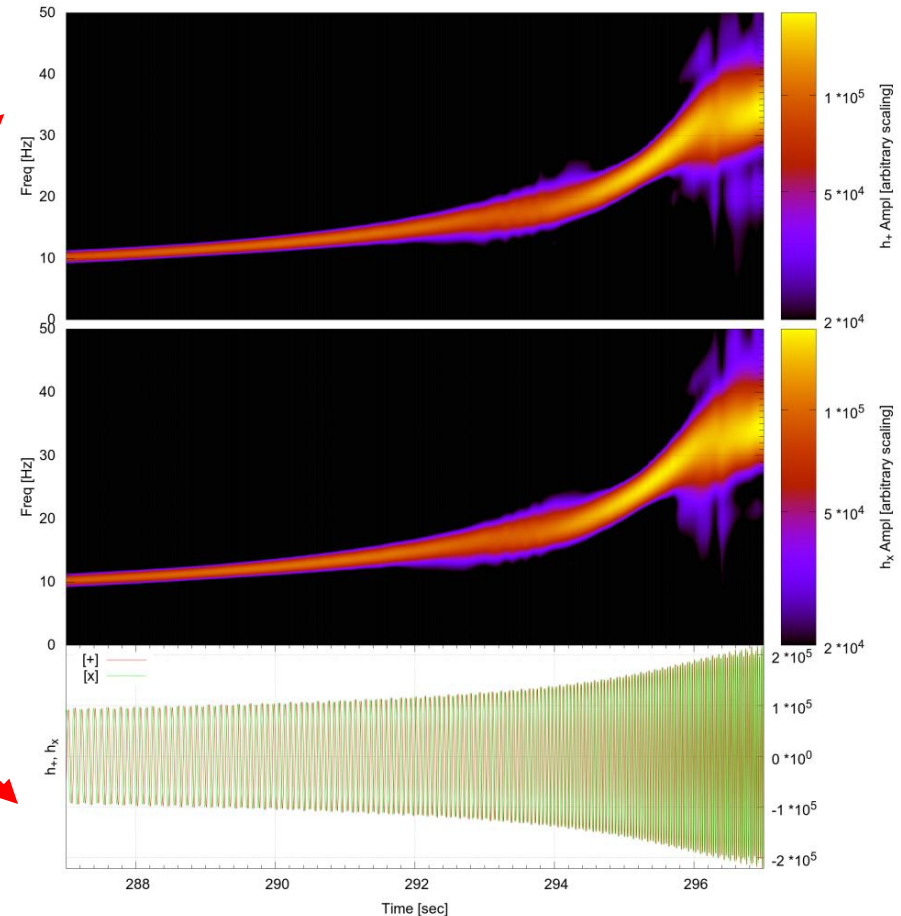
**-Astrophysical results of BBH evolution in
galactic GC's...**

The DRAGON simulations: globular cluster evolution with a million stars

Long Wang,^{1,2★} Rainer Spurzem,^{3,4,1★} Sverre Aarseth,⁵ Mirek Giersz,⁶ Abbas Askar,⁶ Peter Berczik,^{3,4,7} Thorsten Naab,⁸ Riko Schadow⁸ and M. B. N. Kouwenhoven^{1,2}

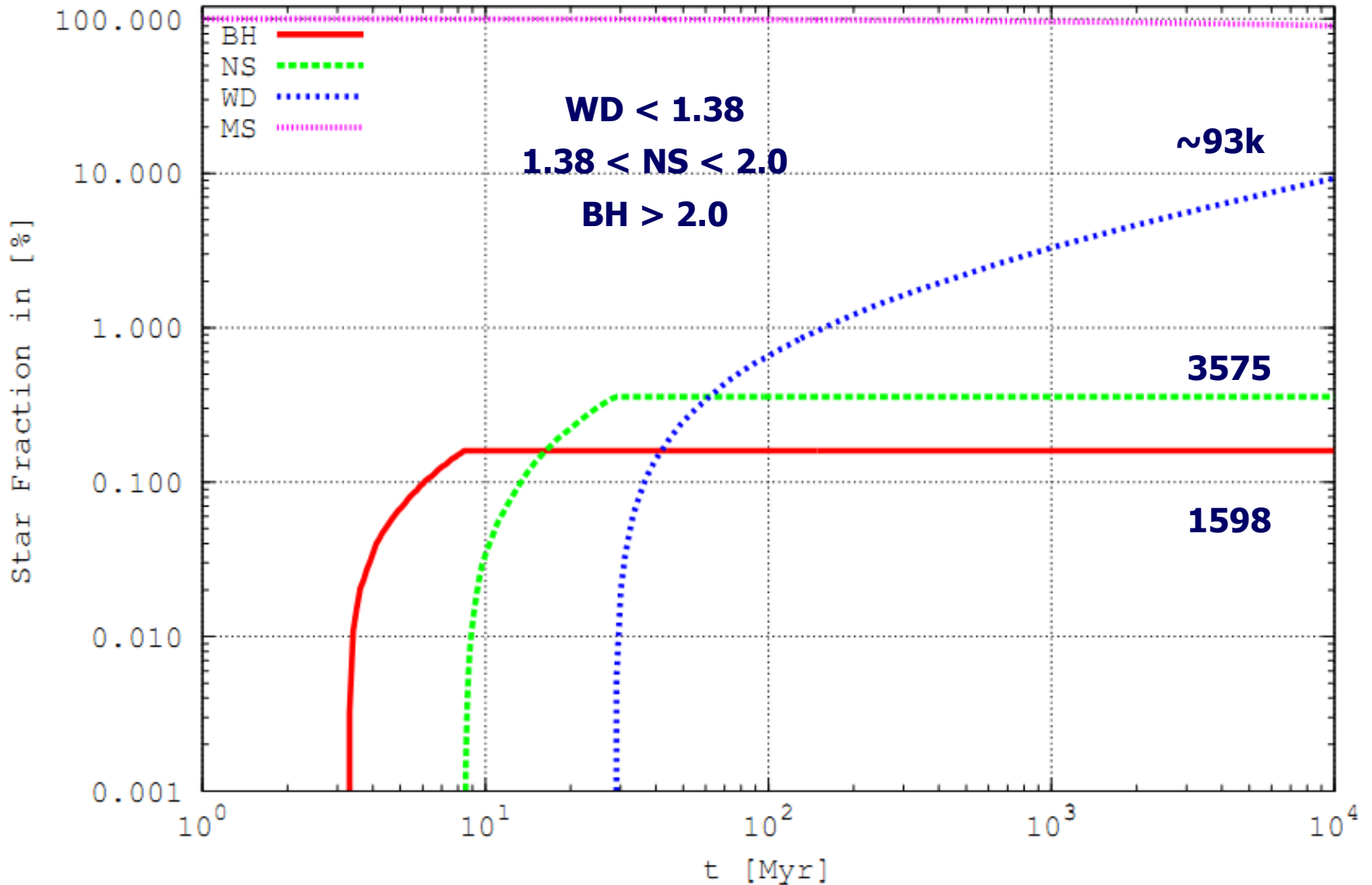


D2-R7-IMF01



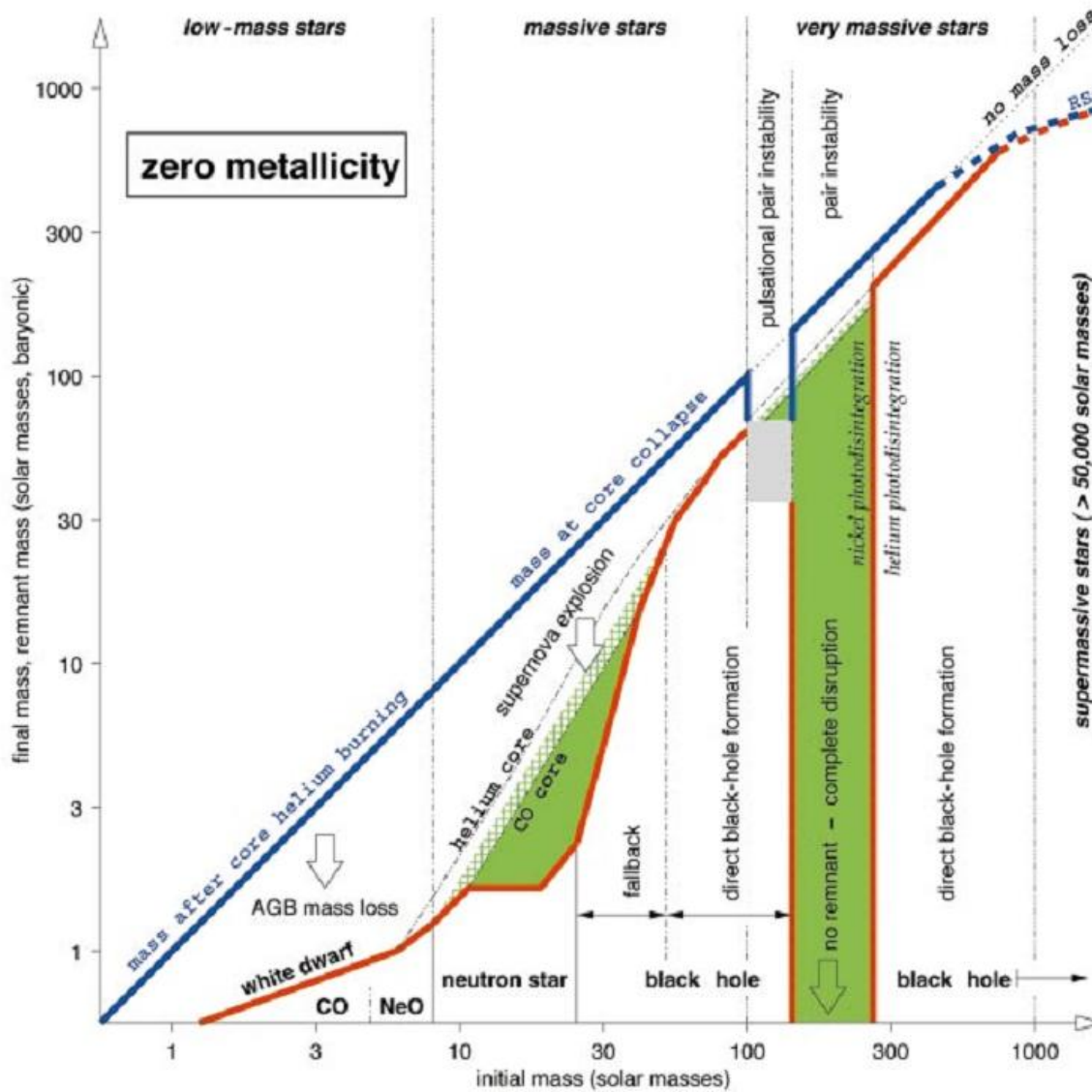
IMF: K2001, 0.08-100.0, N=1M, Z=0.02

~902k



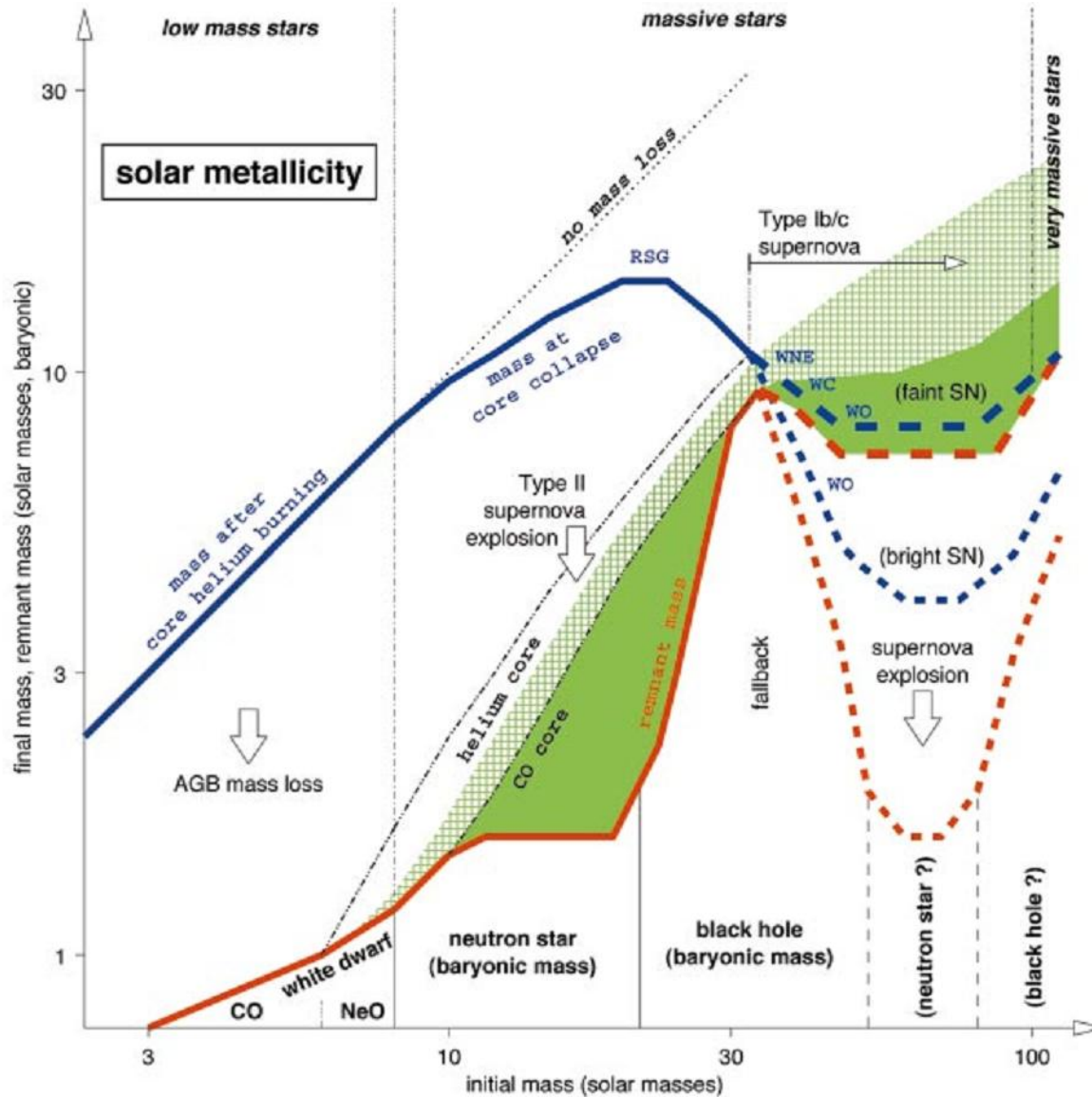
The evolution and explosion of massive stars

T. A. Weaver



The evolution and explosion of massive stars

T. A. Weaver



SSE: Hurley, Onno, Tout, 2000; BSE: 2002

The DRAGON simulations: globular cluster evolution with a million stars

Long Wang,^{1,2★} Rainer Spurzem,^{3,4,1★} Sverre Aarseth,⁵ Mirek Giersz,⁶ Abbas Askar,⁶ Peter Berczik,^{3,4,7} Thorsten Naab,⁸ Riko Schadow⁸ and M. B. N. Kouwenhoven^{1,2}

	D1- R7-IMF93	D2- R7-IMF01	D3- R7-ROT	D4- R3-IMF01
Profile	KW6 ^a	KW6	ES6 ^b	KW6
$R_{h,0}$ (pc)	7.5	7.6	8.1	3.0
IMF	IMF93 ^c	IMF01 ^d	IMF01	IMF01
q	RP ^e	K ^f	K	K
Kick	Low ^g	High ^h	High	High
$R_{t,0}$ (pc)	89	97	97	97

^gLow: $\sigma_k \approx 30 \text{ km s}^{-1}$. Hobbs et al. (2005)

^hHigh: $\sigma_k = 265 \text{ km s}^{-1}$.

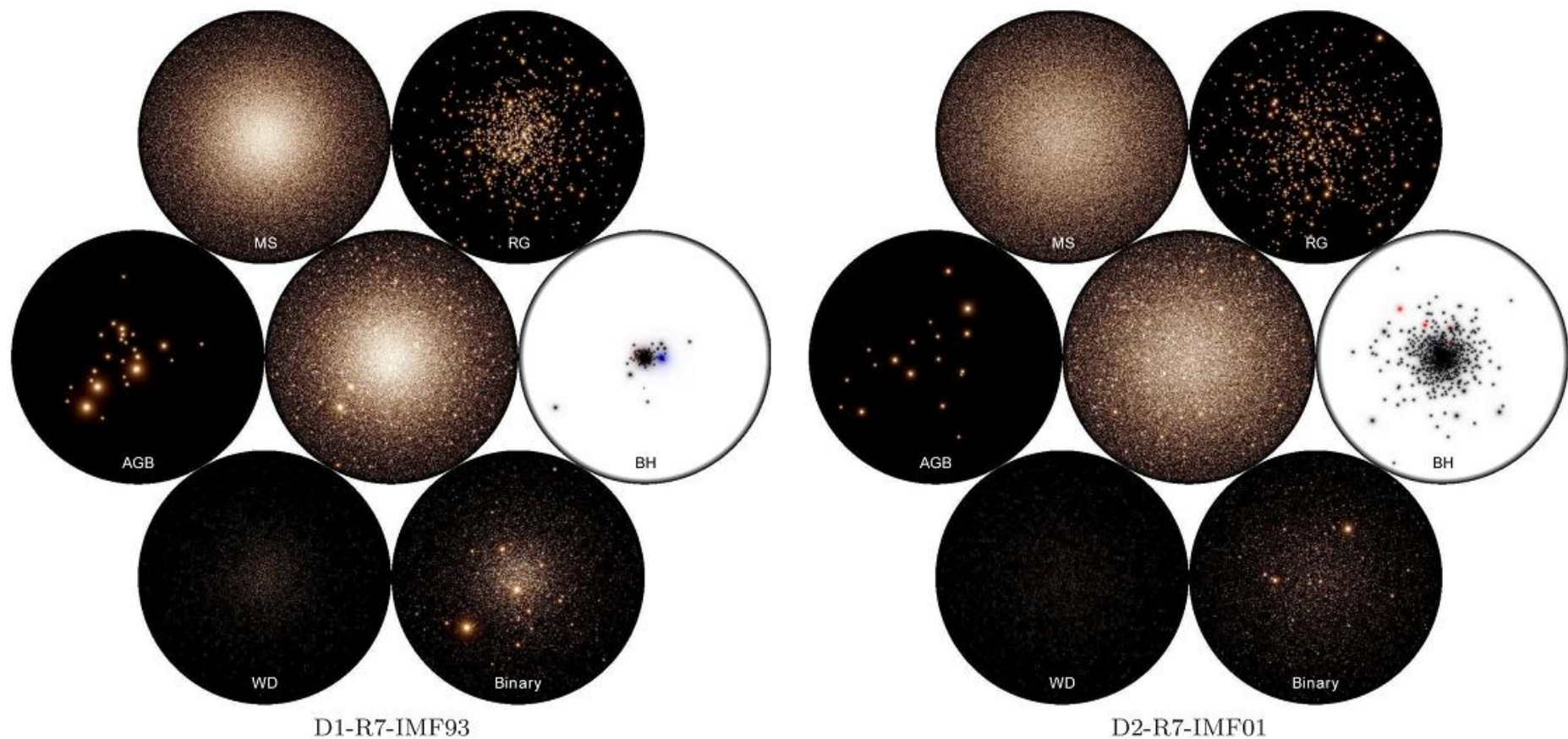
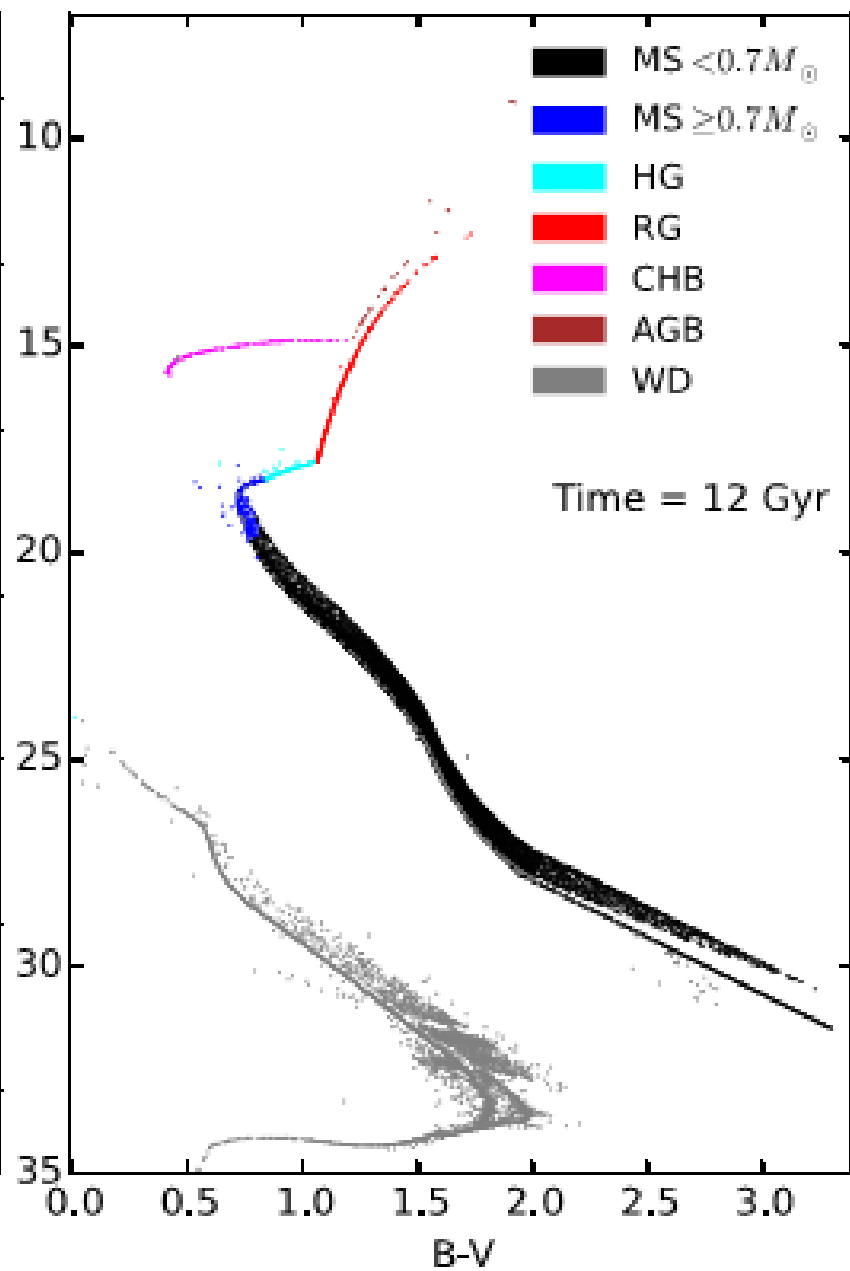
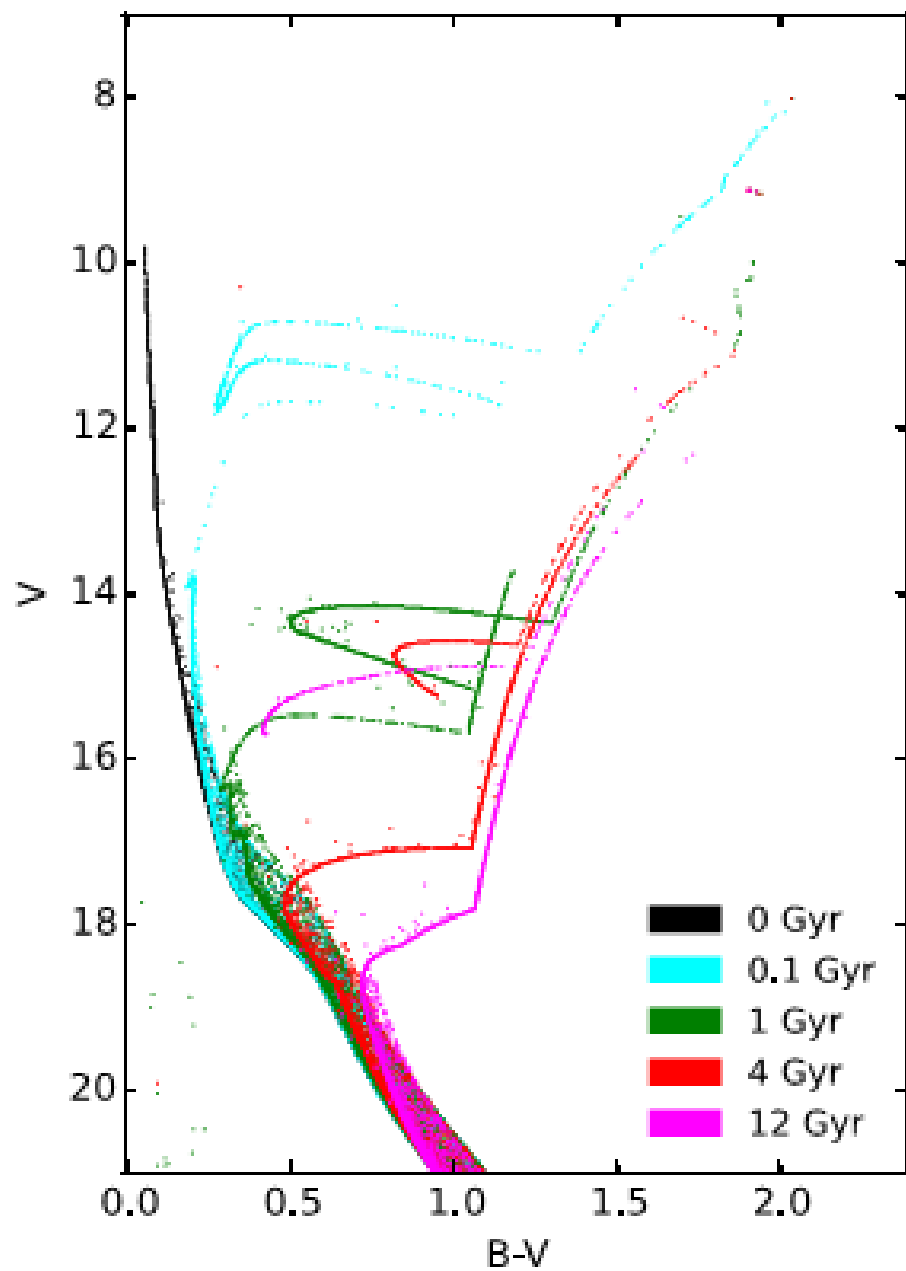


Figure 1. Snapshot of the D1-R7-IMF93 and D2-R7-IMF01 models at 12 Gyr as observed in B , V and I . The diameter of each image is 57.6 pc with 2048×2048 pixels ($1.0 \text{ arcsec pixel}^{-1}$). The Moffat (1969) point spread function is used here with seeing of 0.5 arcsec. There is no background fluctuation, field stars contamination or cosmic rays. The colours are generated by B (blue), V (green) and I (red) bands. The brightness is shown on a logarithm scale. The different types of stars are also shown individually. The labels MS, RG, AGB, WD and BH represent main sequence, red giant, asymptotic giant branch, white dwarf and BH, respectively. The exposure time of WDs is enlarged by a factor of 10^4 . The dot size in the BH panel is proportional to the BH mass; the red dots represent binaries with one BH component and the blue points are BH–BH binaries (due to crowding, particularly in the central regions, some binaries are overlapped by single BHs).



	D1- R7-IMF93	D2- R7-IMF01	D3- R7-ROT	D4- R3-IMF01
Profile	KW6 ^a	KW6	ES6 ^b	KW6
$R_{h,0}$ (pc)	7.5	7.6	8.1	3.0
IMF	IMF93 ^c	IMF01 ^d	IMF01	IMF01
q	RP ^e	K ^f	K	K
Kick	Low ^g	High ^h	High	High
$R_{t,0}$ (pc)	89	97	97	97

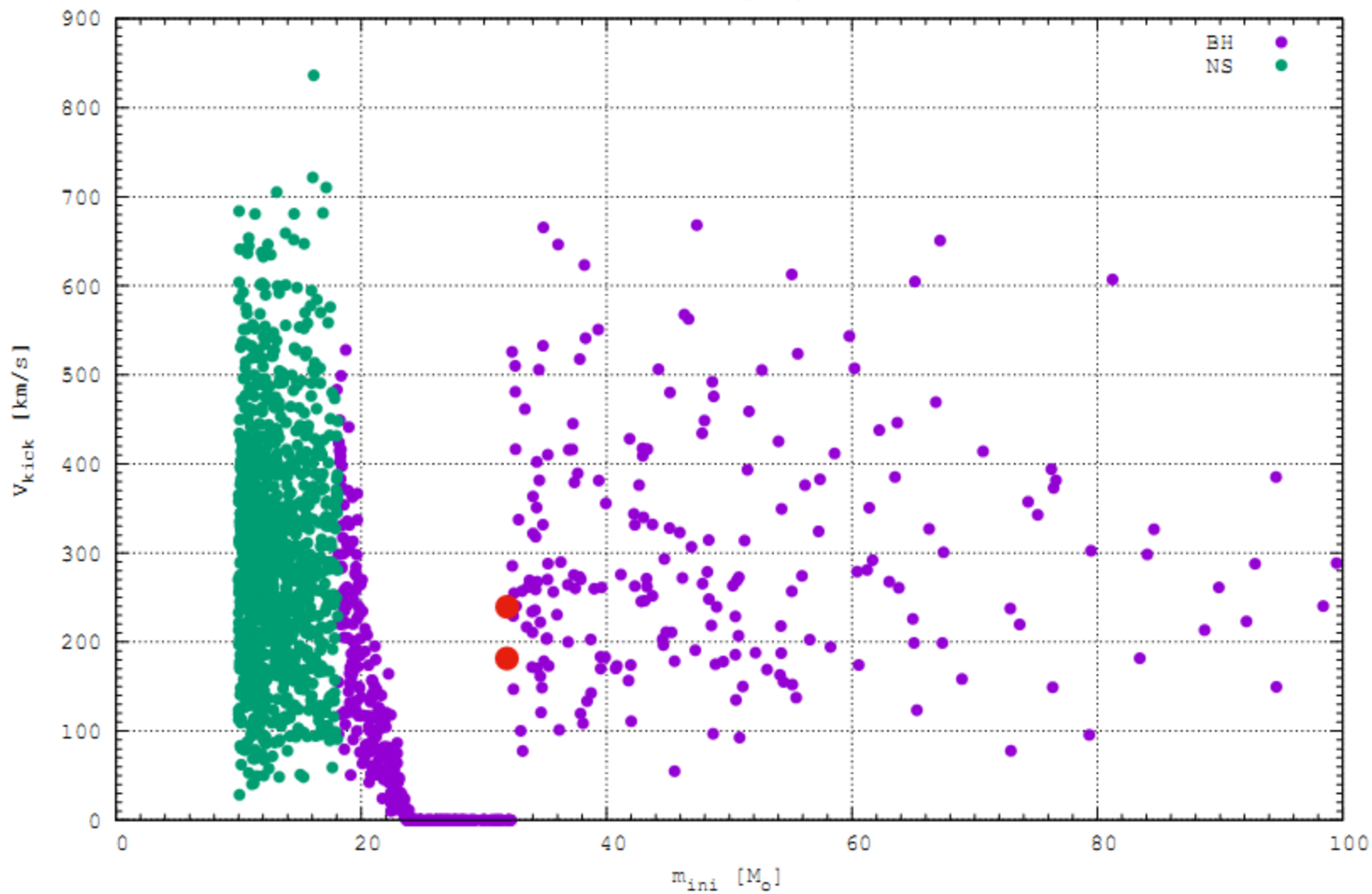
Time (Myr)	D1-R7-IMF93			D2-R7-IMF01			D3-R7-ROT			D4-R3-IMF01		
	WD	NS	BH	WD	NS	BH	WD	NS	BH	WD	NS	BH
50	0	420	438	0	104	1436	0	106	1432	0	68	1458
100	53	172	435	3232	0	1435	3228	1	1428	3204	3	1458
500	15 254	150	433	20 783	0	1421	20 642	0	1420	20 821	2	1401
1000	25 681	143	425	32 302	0	1409	31 914	0	1414	32 705	2	1329
2000	43 080	137	401	49 311	0	1352	49 237	0	1374			
4000	62 740	130	348	66 306	0	1258	68 570	0	1282			
6000	75 916	125	305	75 836	0	1180	80 522	0	1220			
8000	85 323	125	281	81 215	0	1130	88 659	0	1172			
10000	92 572	124	262	84 354	0	1080	94 250	0	1130			
12000	98 563	121	245	85 614	0	1037	97 824	0	1096			
ALL	104 175	2942	629	105 652	6697	2096	107 587	6696	2096	32732	6736	2092

^gLow: $\sigma_k \approx 30 \text{ km s}^{-1}$.

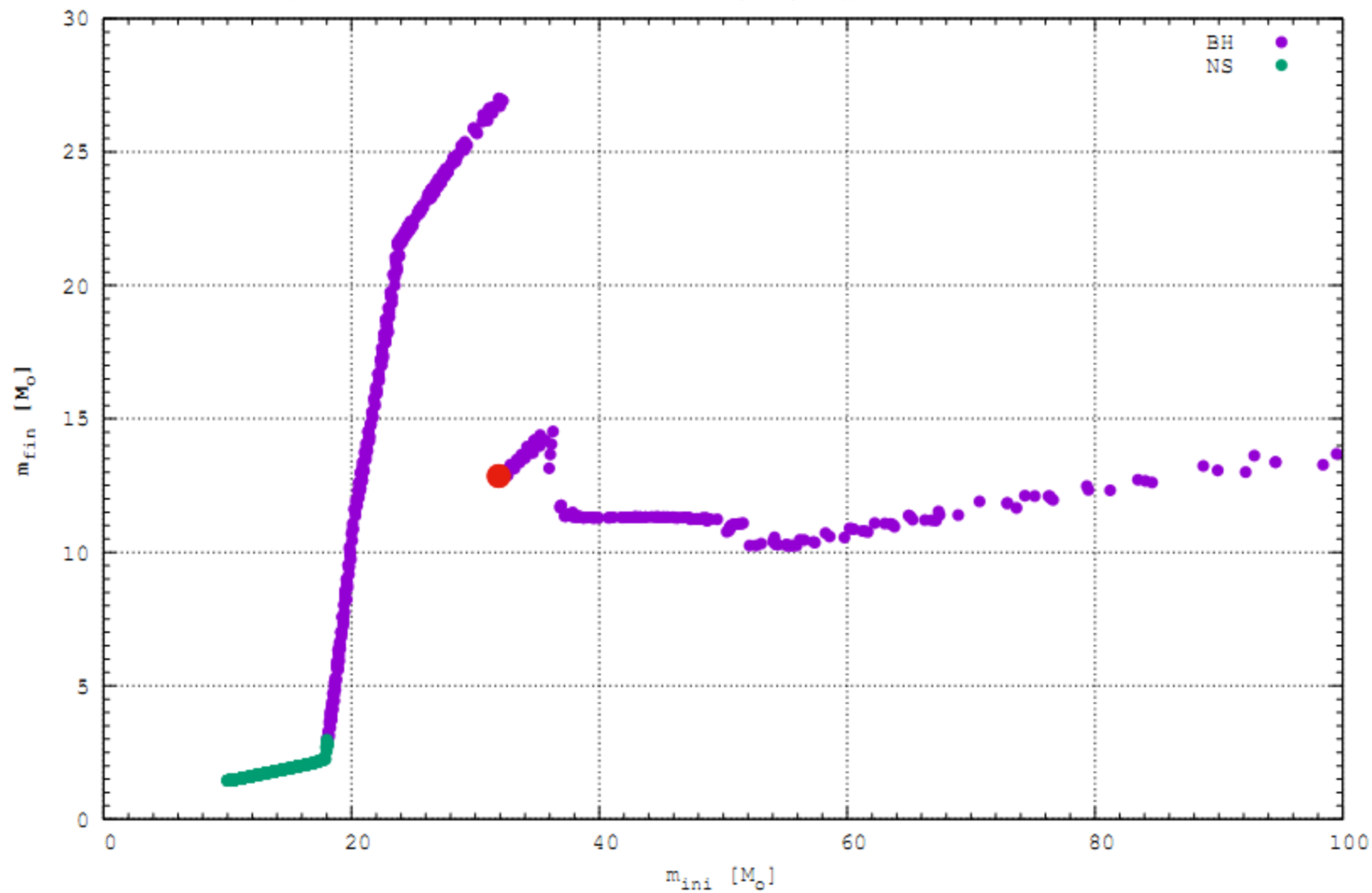
^hHigh: $\sigma_k = 265 \text{ km s}^{-1}$.

Hobbs et al. (2005)

N=1050k, King, $W_0=6$, Φ_{MW} , $R_0=7.1\text{kpc}$, $M=4.76 \cdot 10^5 M_\odot$, $R_J/R_{100}=1.8$, $R_{hm}=7.5\text{pc}$, $RU=9.32\text{pc}$, $TU=0.61\text{Myr}$

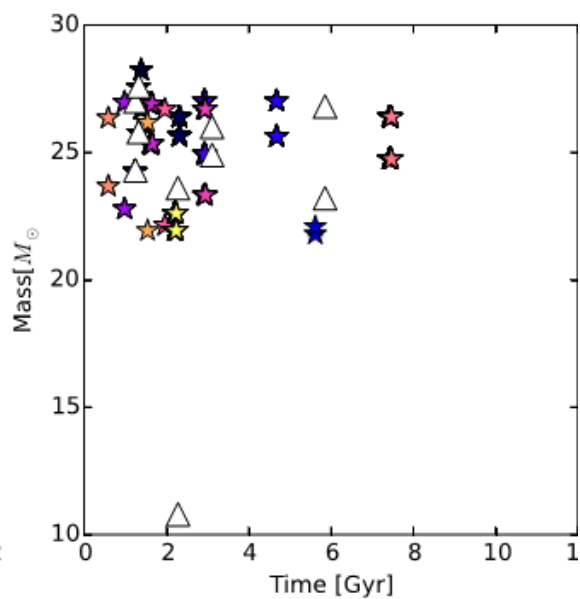
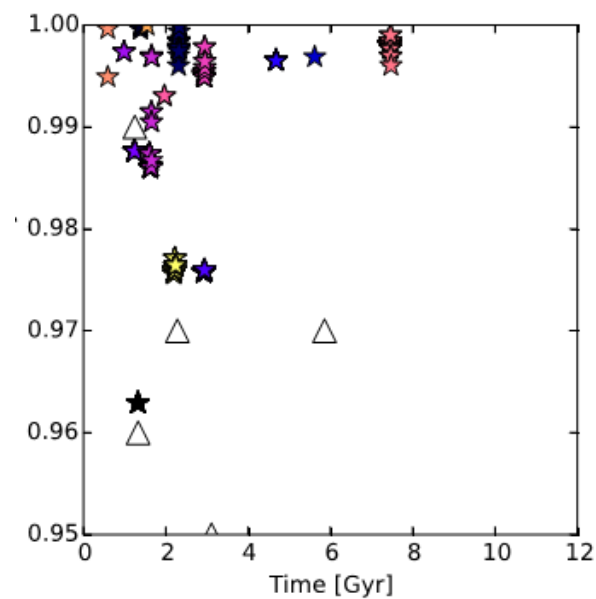
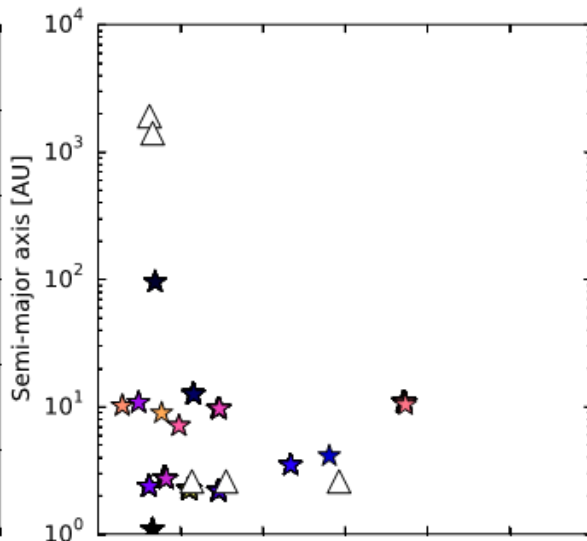
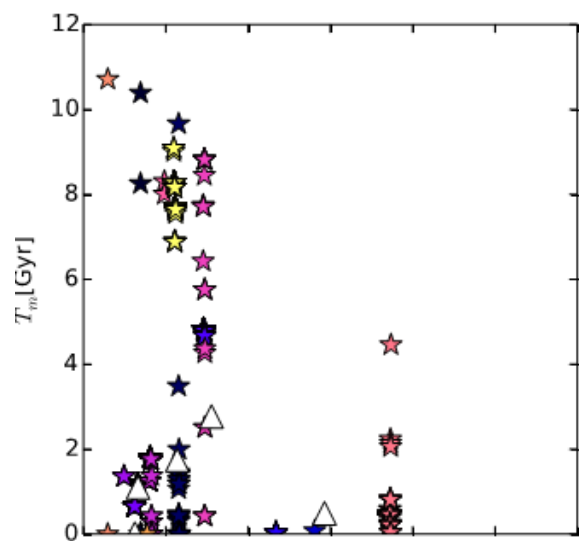


$N=1050k$, King, $W_0=6$, Φ_{MW} , $R_0=7.1\text{kpc}$, $M=4.76 \cdot 10^5 M_\odot$, $R_J/R_{100}=1.8$, $R_{hm}=7.5\text{pc}$, $RU=9.32\text{pc}$, $TU=0.61\text{Myr}$



★ ★ BH binaries in GCs

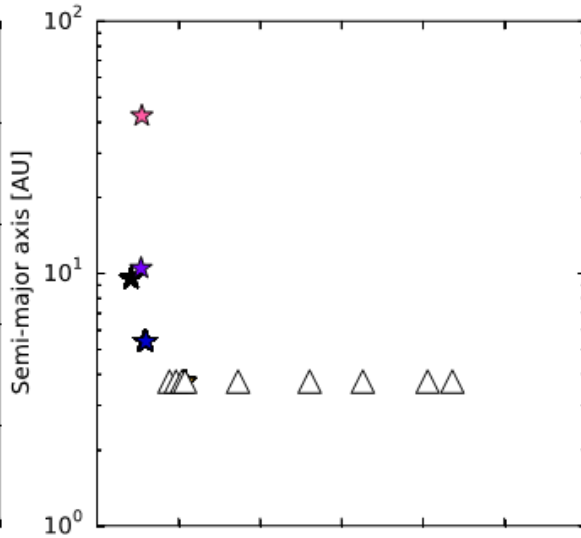
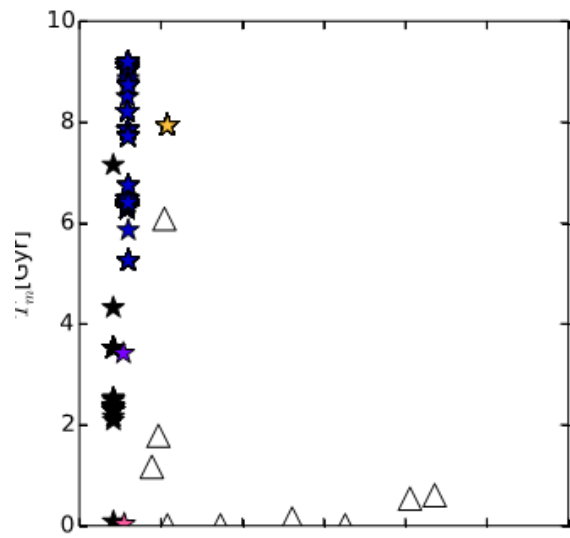
▲ ▲ BH binaries escapers



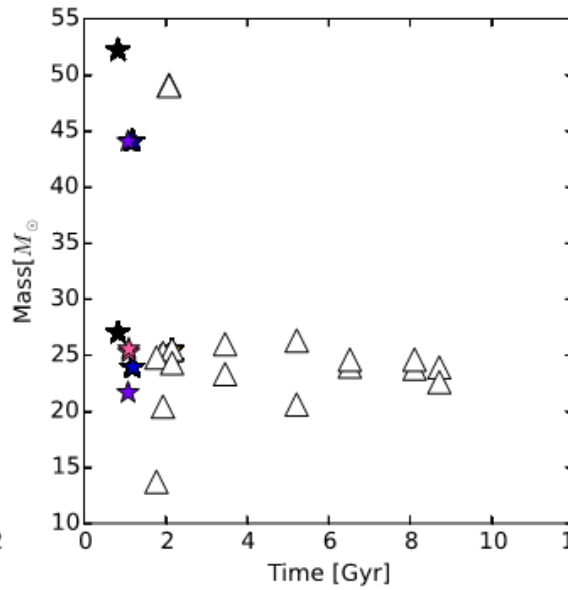
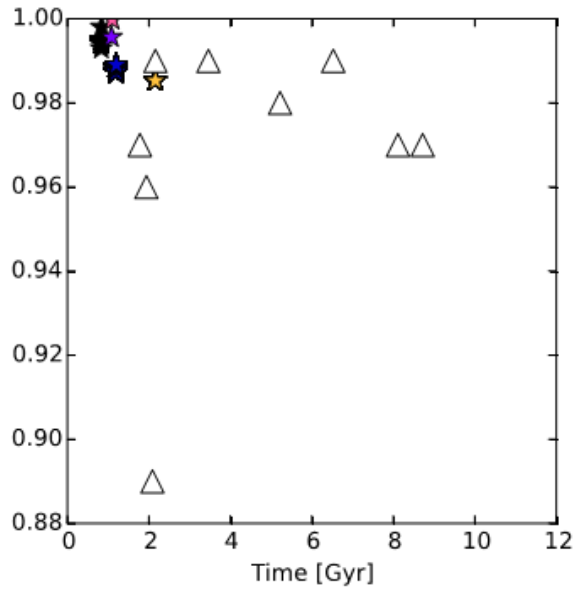
D1-
R7-IMF93

* * BH binaries in GCs

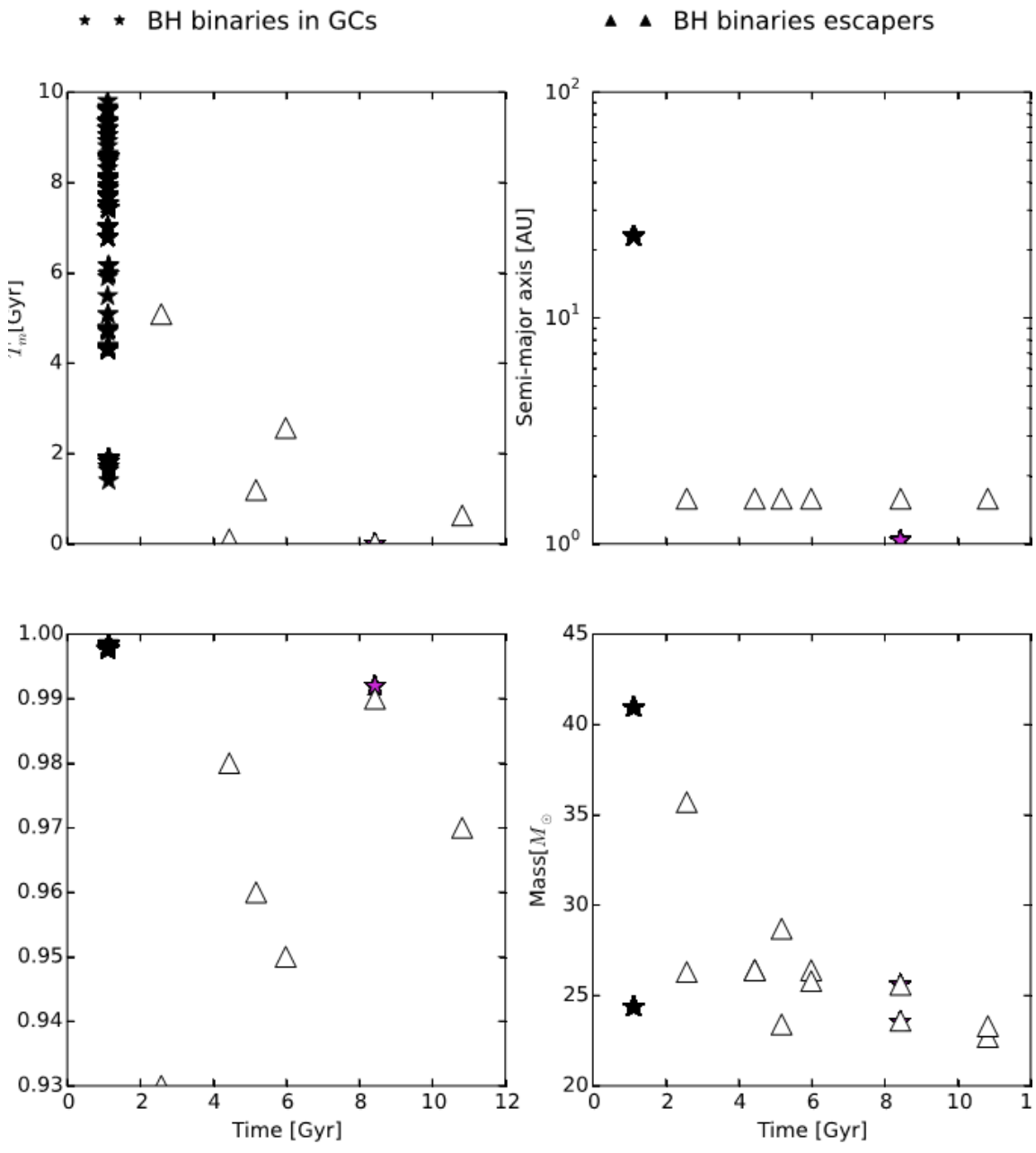
▲ ▲ BH binaries escapers



D2-
R7-IMF01

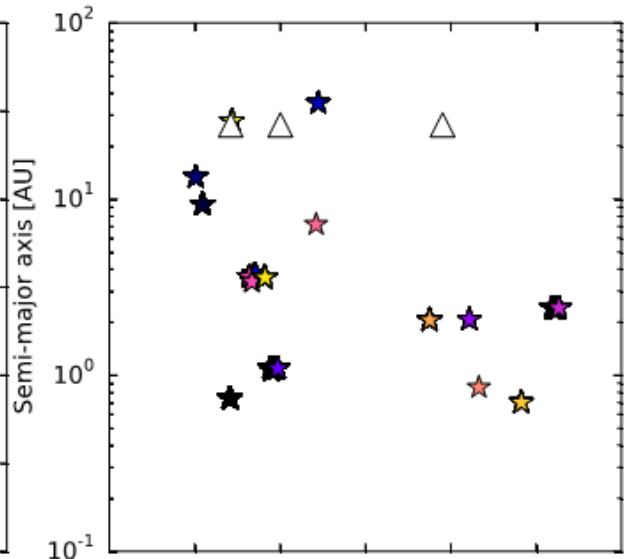
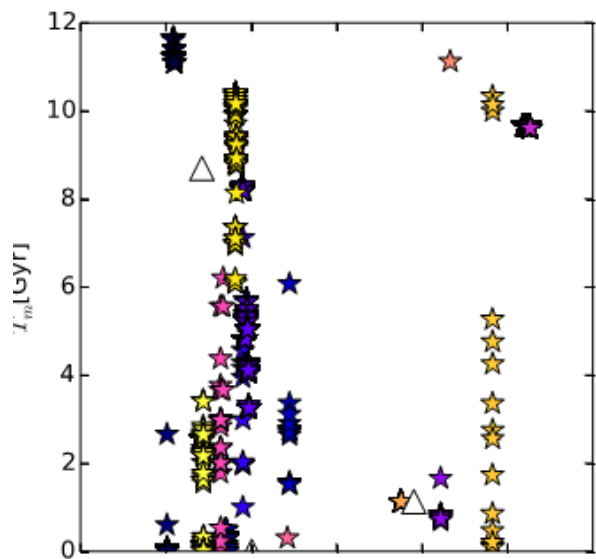


D3-
R7-ROT

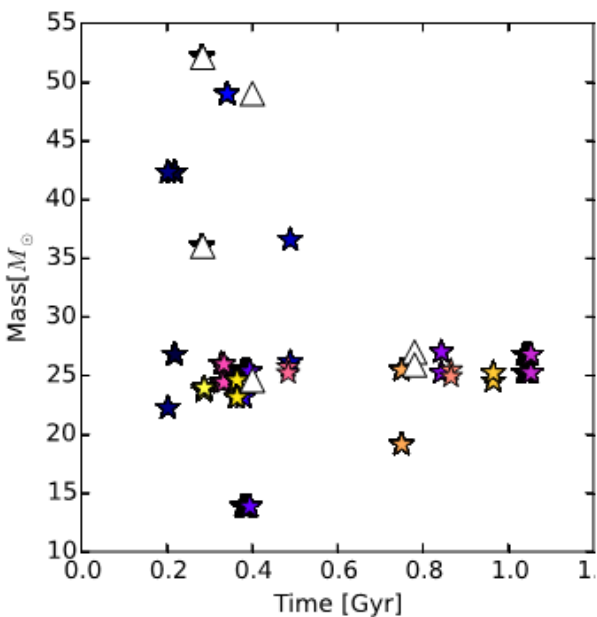
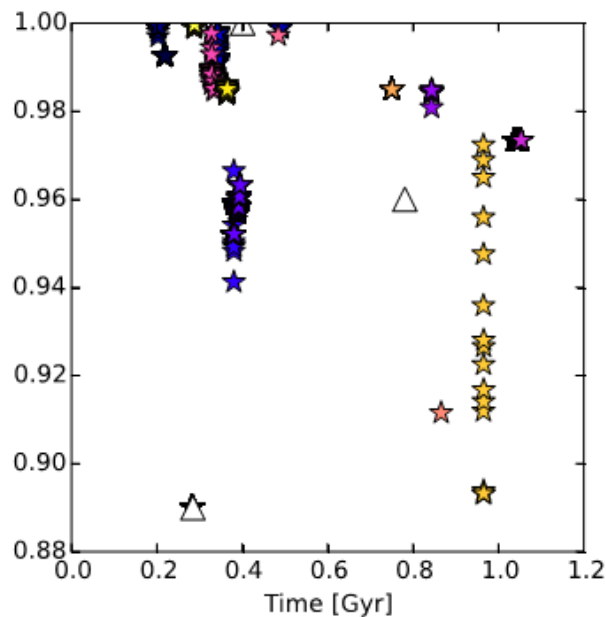


★ ★ BH binaries in GCs

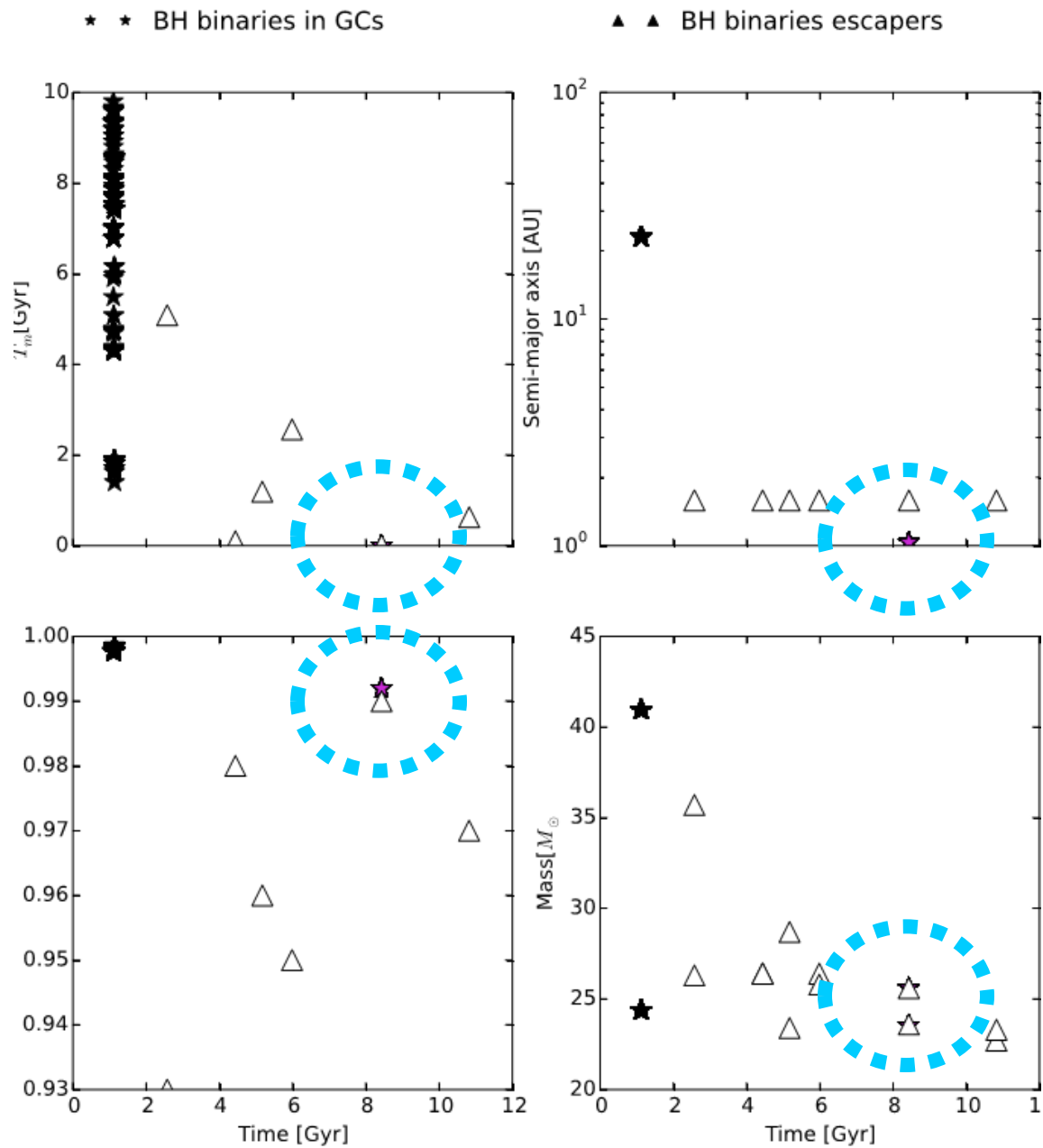
▲ ▲ BH binaries escapers



D4-
R3-IMF01



D3-
R7-ROT



```

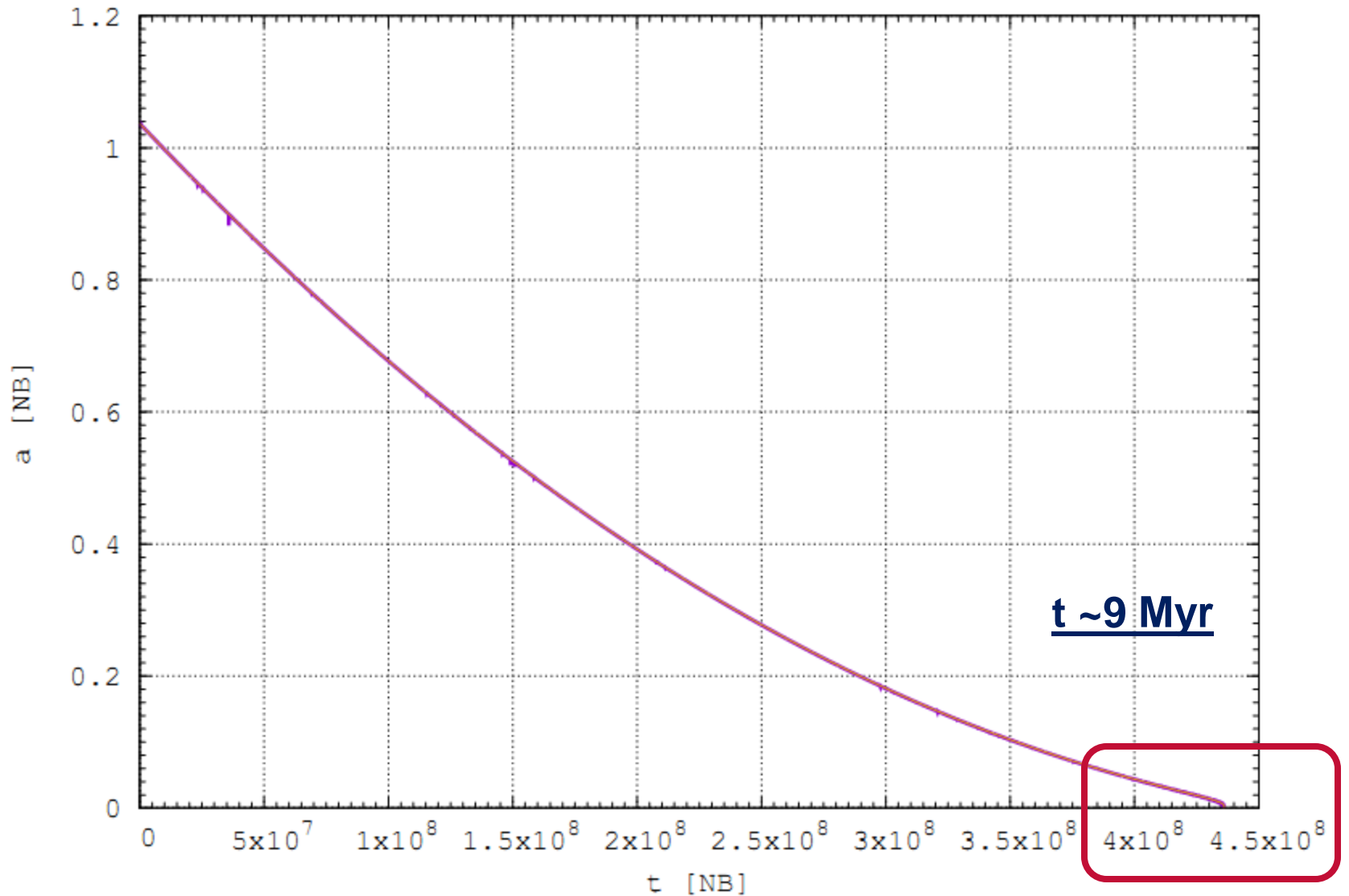
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V_NB = 2.10613574E+02 [km/s]    T_NB = 2.25083864E-02 [yr]
=====
=====
c      = 1.42342420E+03 [NB]      M_BH = 1.00000000E+00 [NB]
R_sw   = 9.87100353E-07 [NB]      = 1.47668111E+02 [km]
T_orb  = 6.28318531E+00 [NB]      = 1.41424363E-01 [yr]
=====
=====
M1     = 2.35554123000000E+01 [Mo]  4.71108246000000E-01 [NB]
M2     = 2.56100902999999E+01 [Mo]  5.12201805999999E-01 [NB]
a      = 1.03684855000000E+00 [AU]  1.03684855000000E+00 [NB]
ecc    = 9.91988301000000E-01
T_GW  = 5.8862377098717E+06 [yr]    2.6151309146132E+08 [NB]
=====
T_GW  = 1.1610939125660E+07 [yr]    5.1584946704215E+08 [NB]
=====

```

101160 – 100975 (+ other ~20 BBH)

Berczik et al. (in prep.)

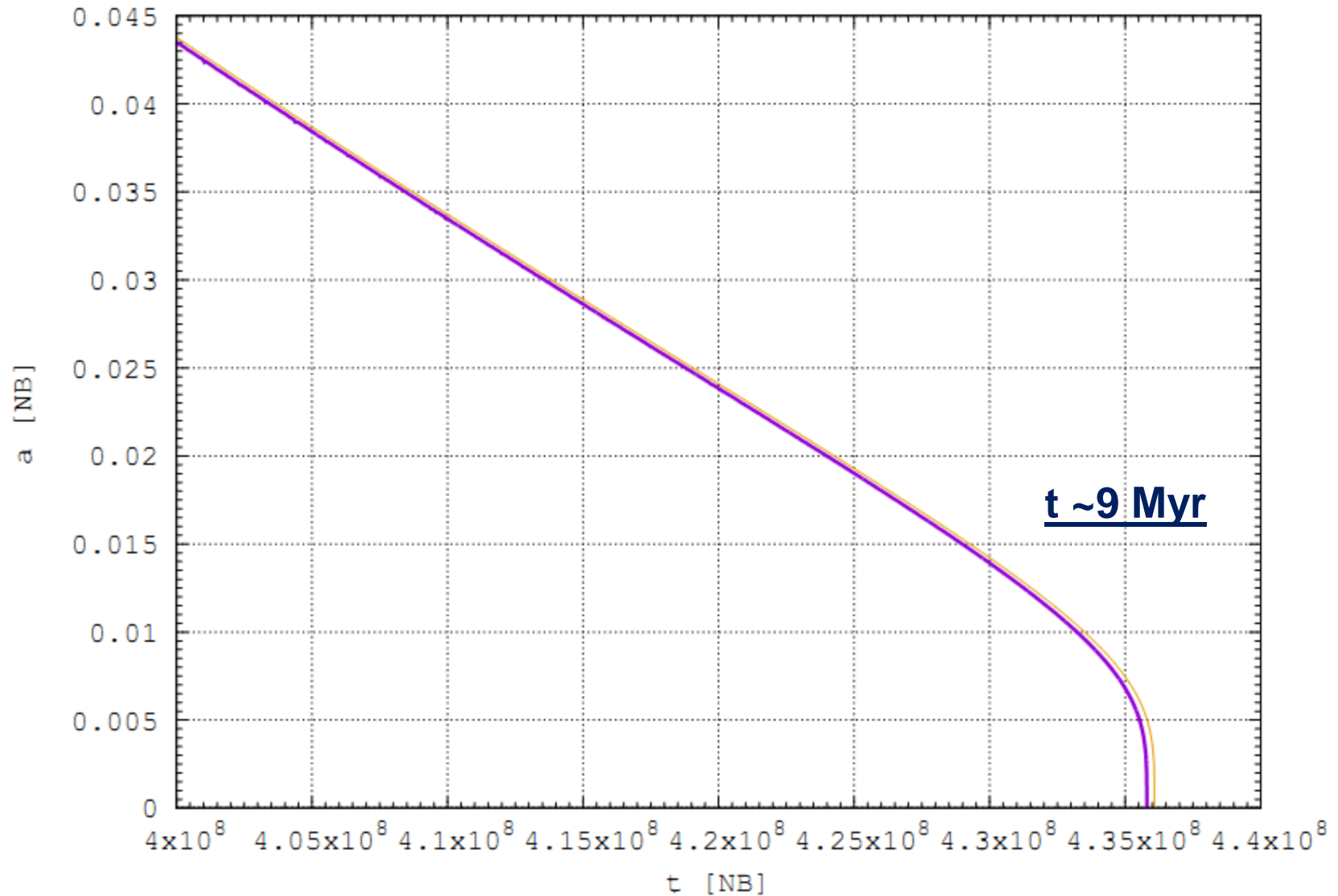
$a = 1 \text{ AU}; t = 0.022 \text{ yr}$



101160 - 100975

Berczik et al. (in prep.)

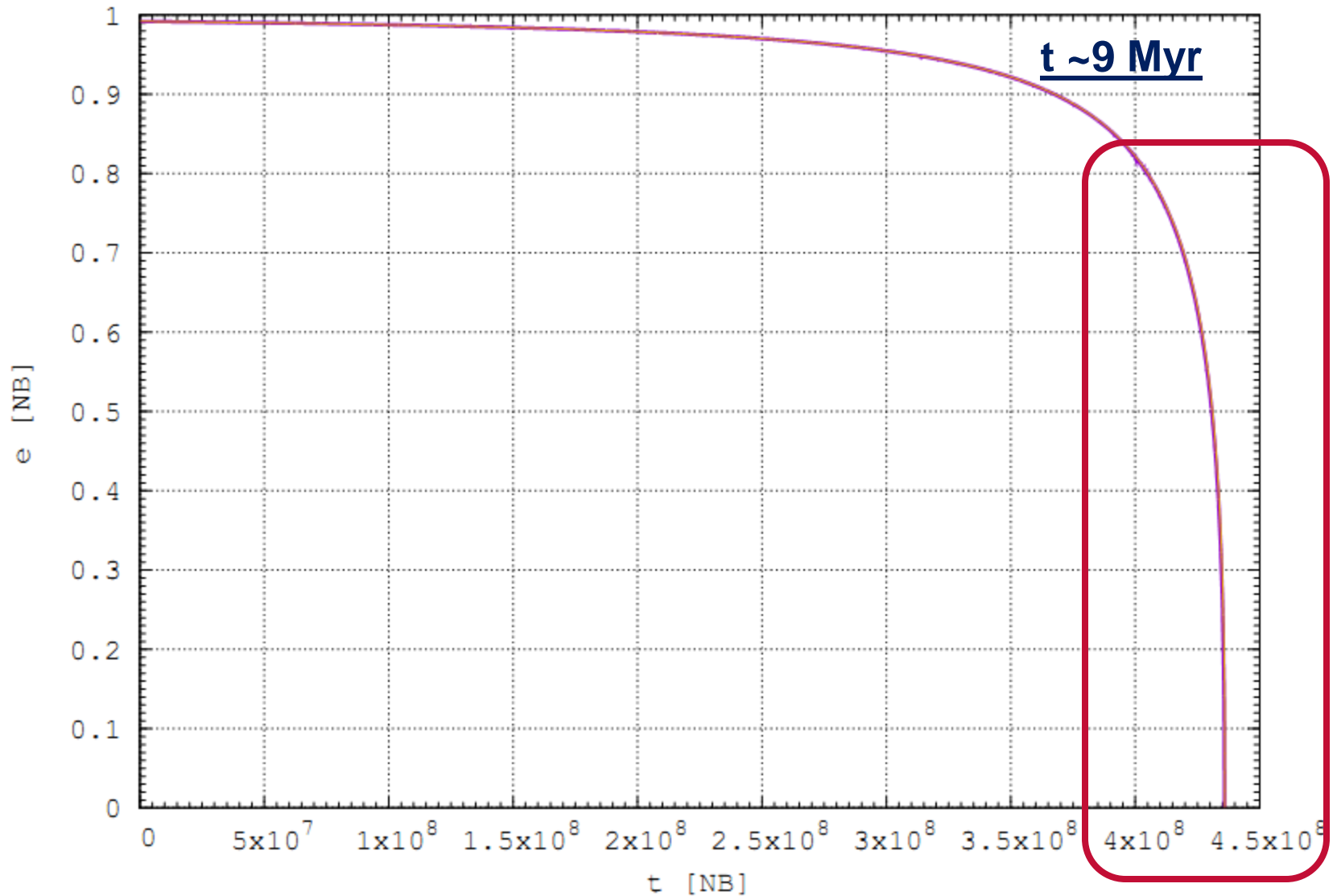
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101160 - 100975

Berczik et al. (in prep.)

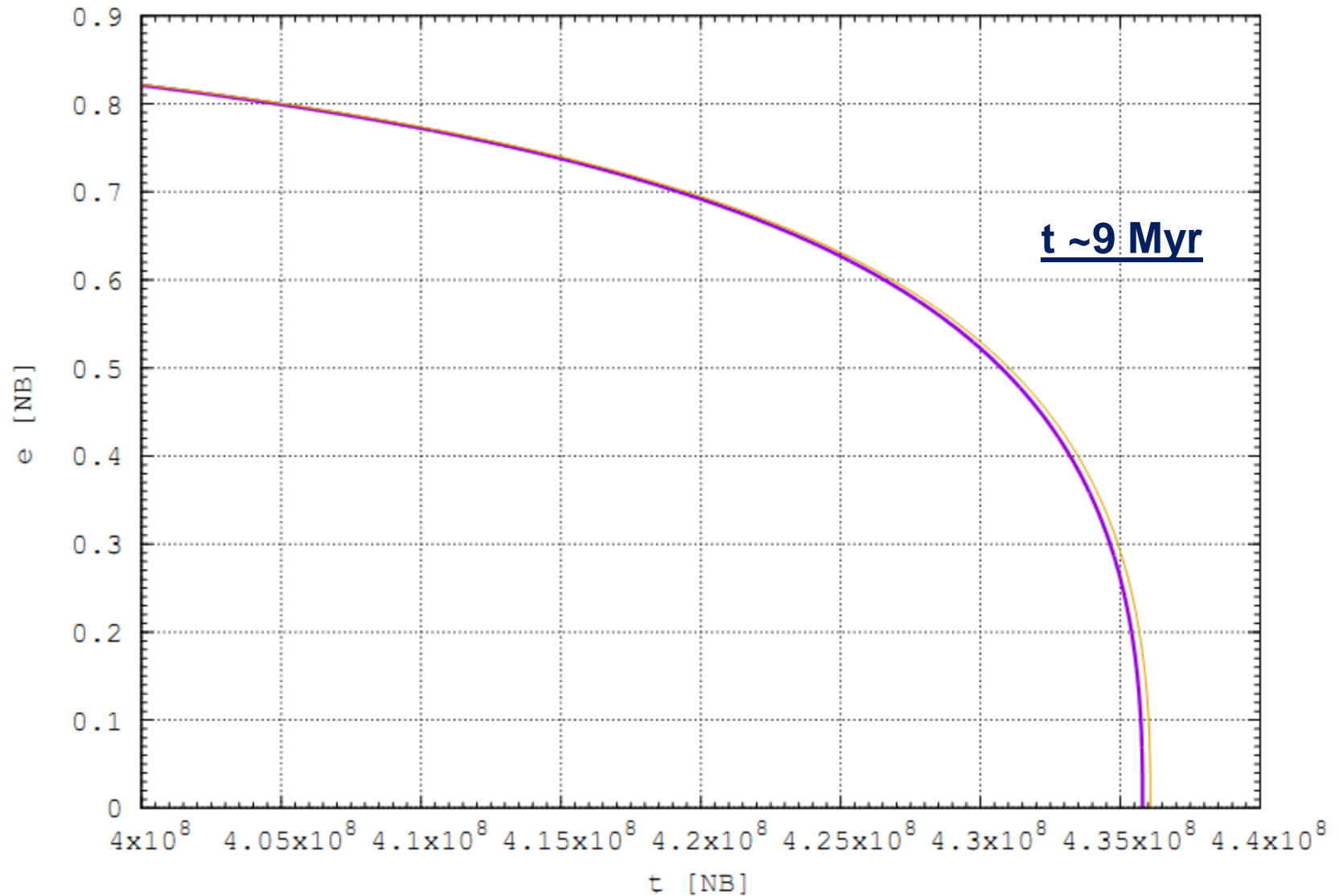
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101160 - 100975

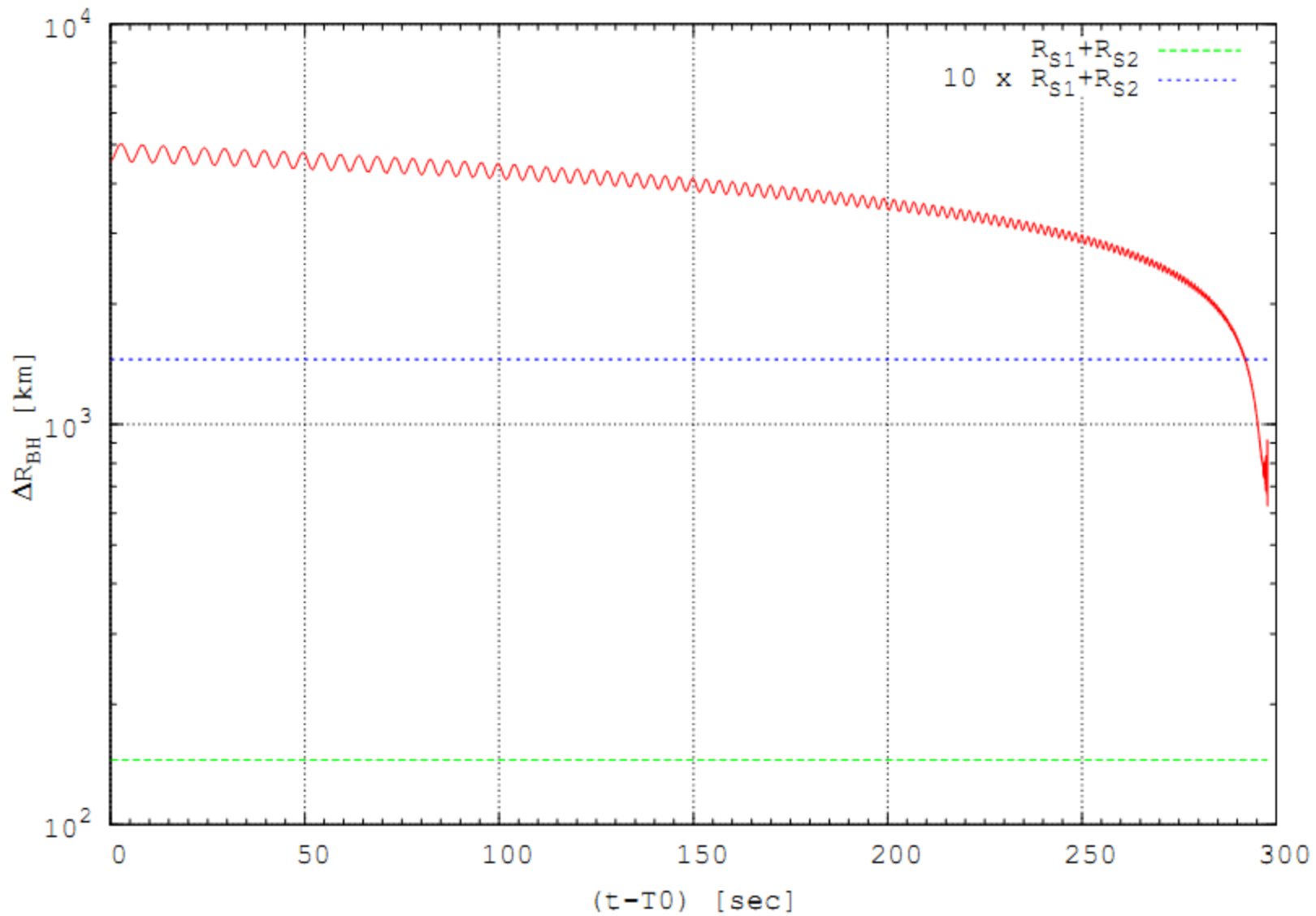
Berczik et al. (in prep.)

$a = 1 \text{ AU}; t = 0.022 \text{ yr}$

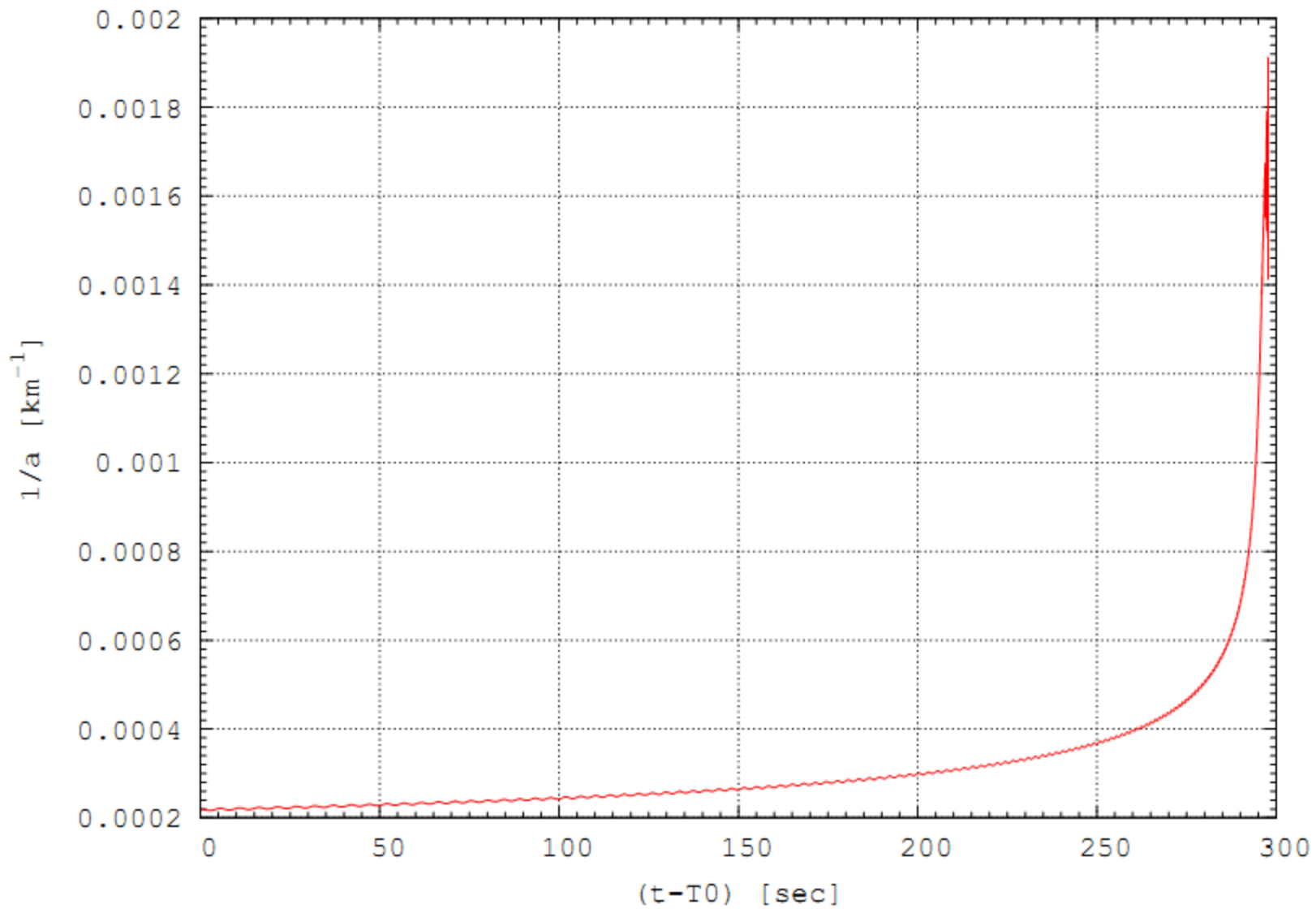


101160 - 100975

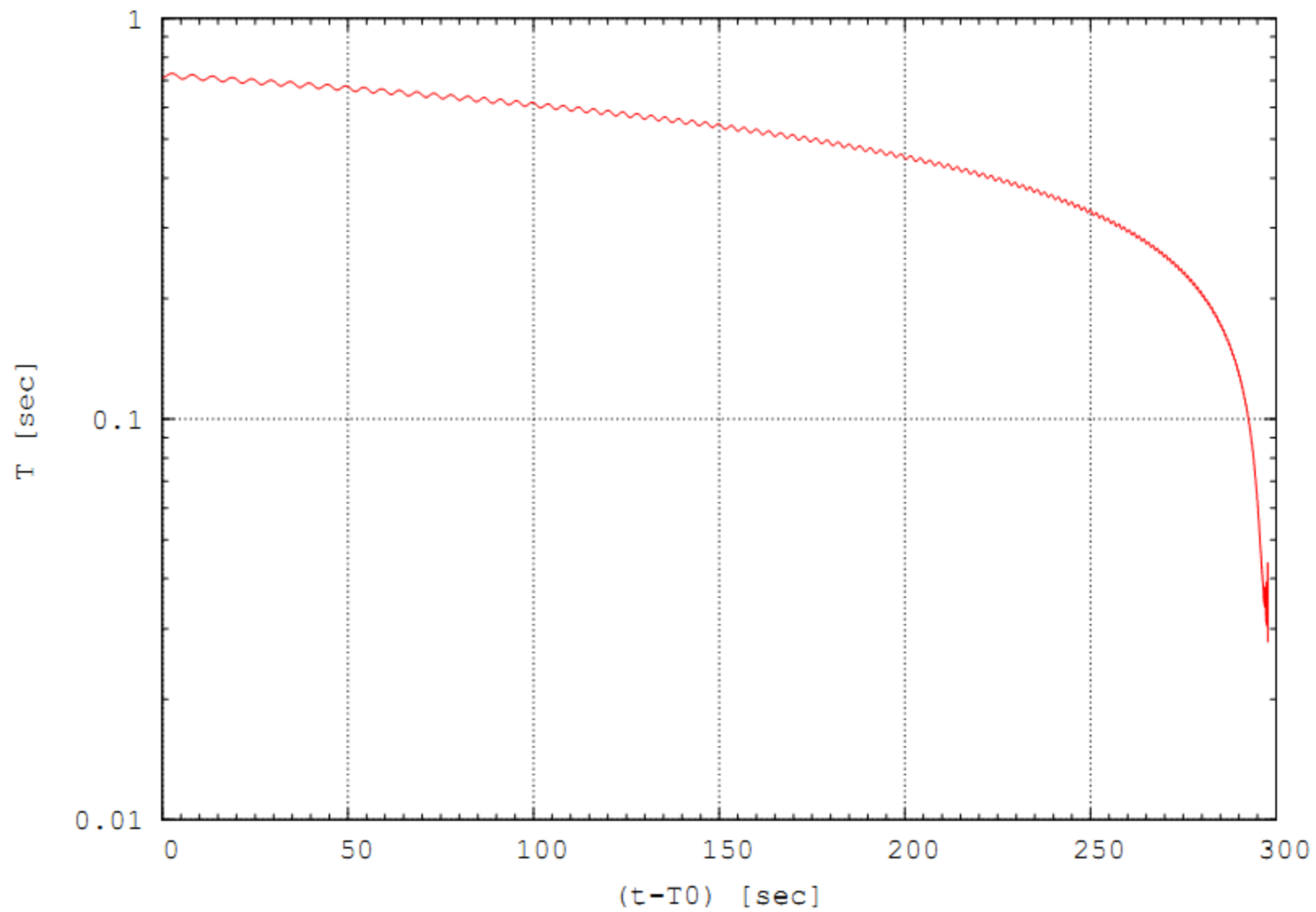
Berczik et al. (in prep.)



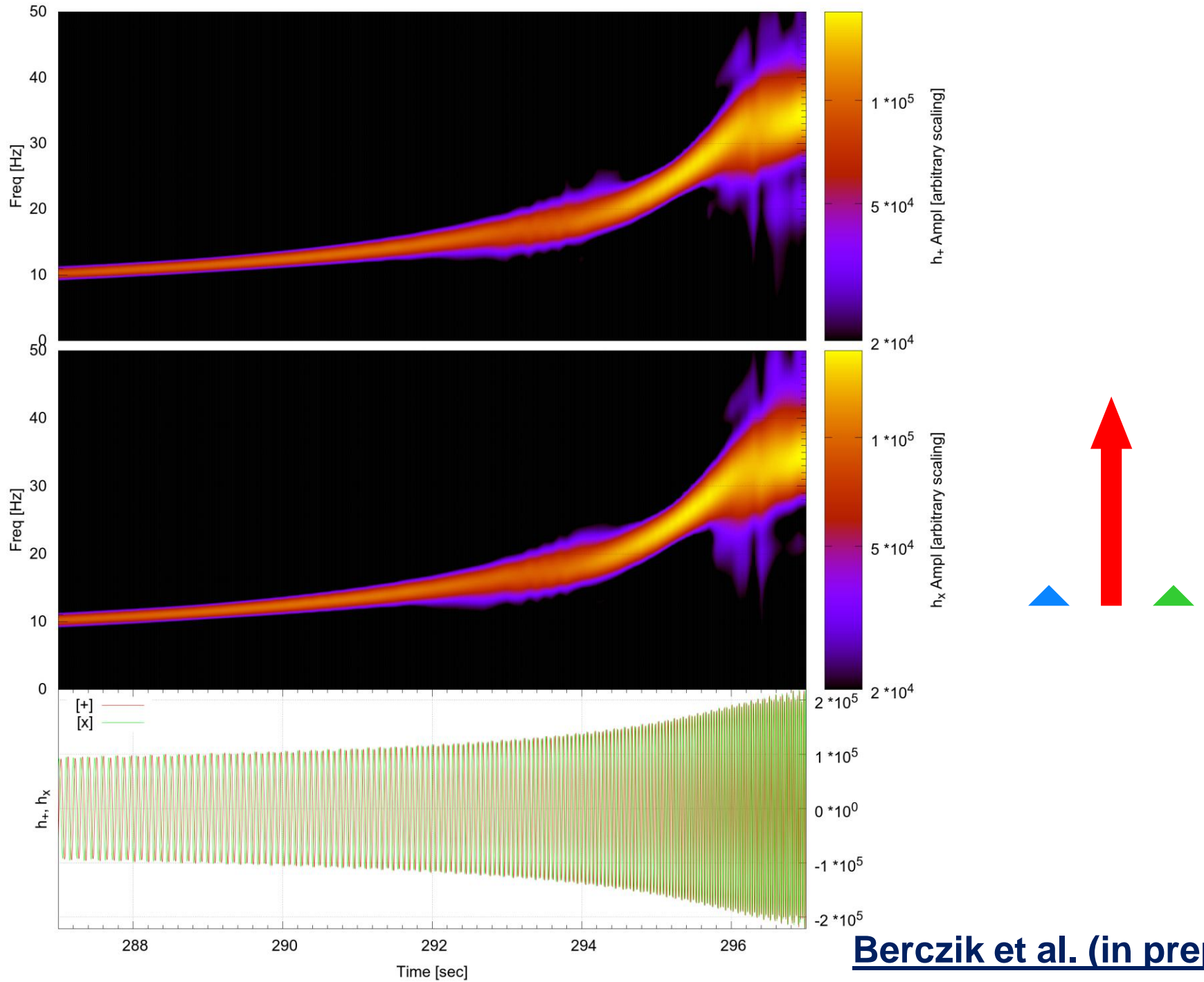
101160 - 100975



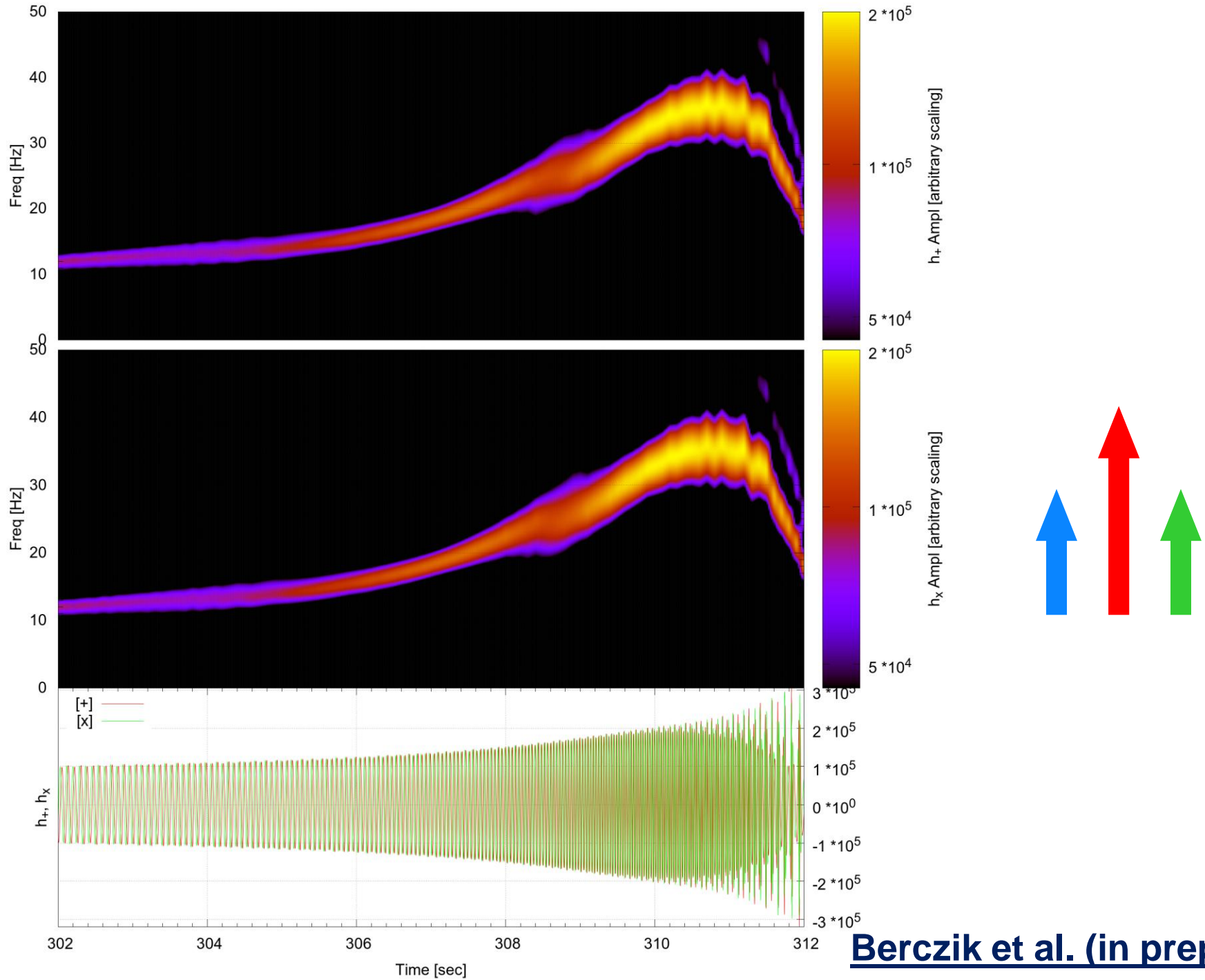
101160 - 100975

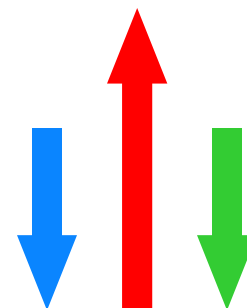
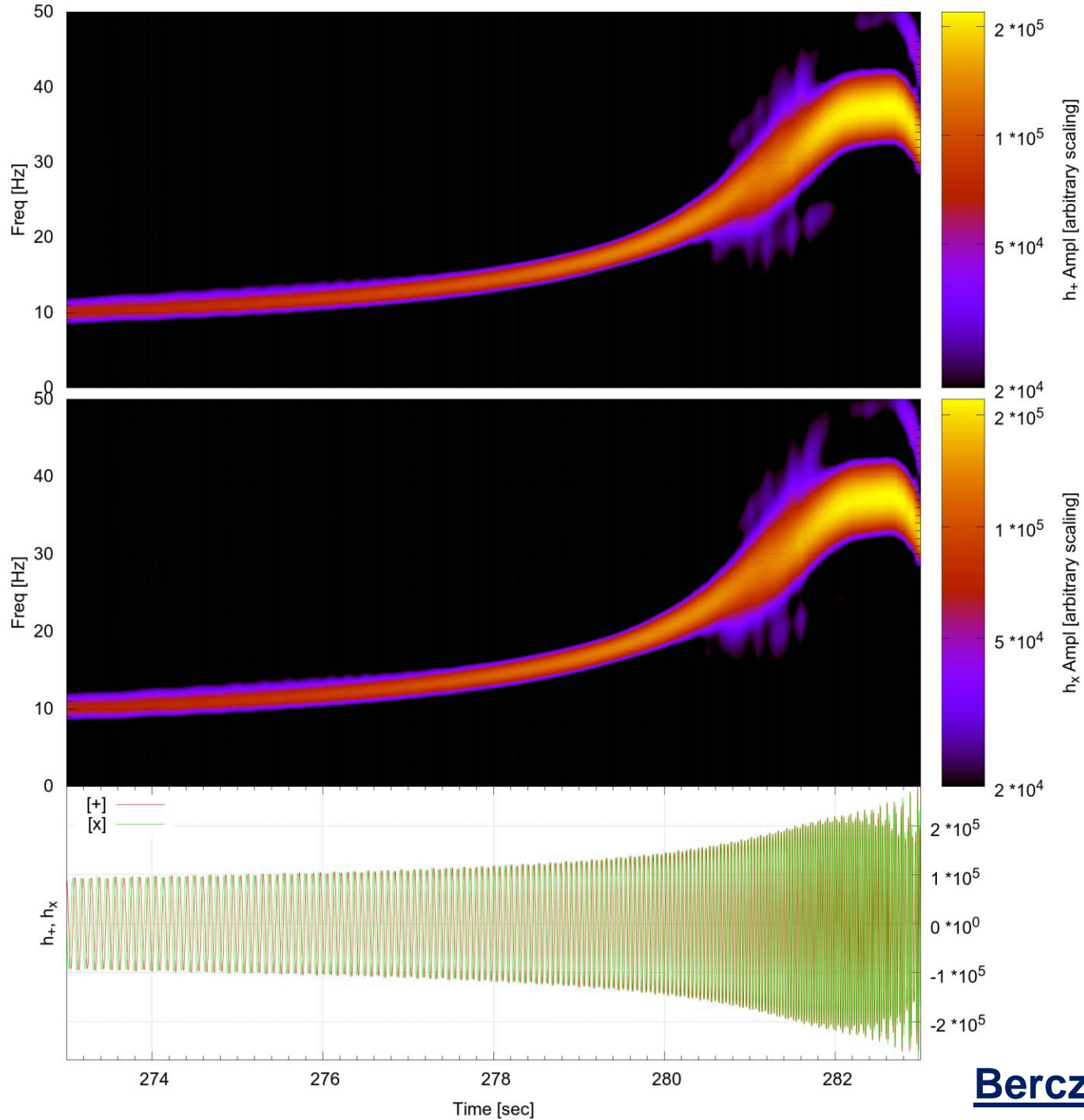


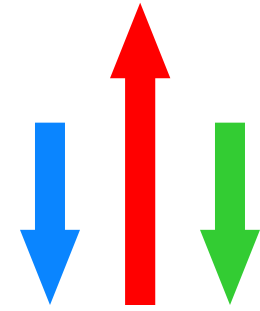
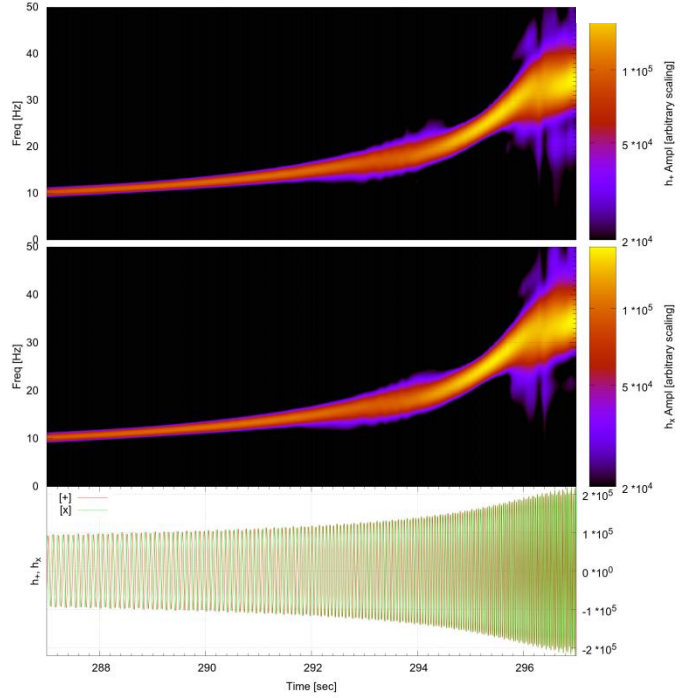
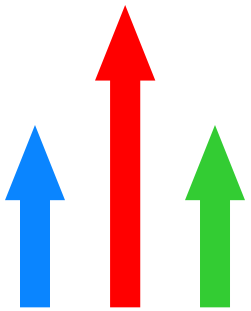
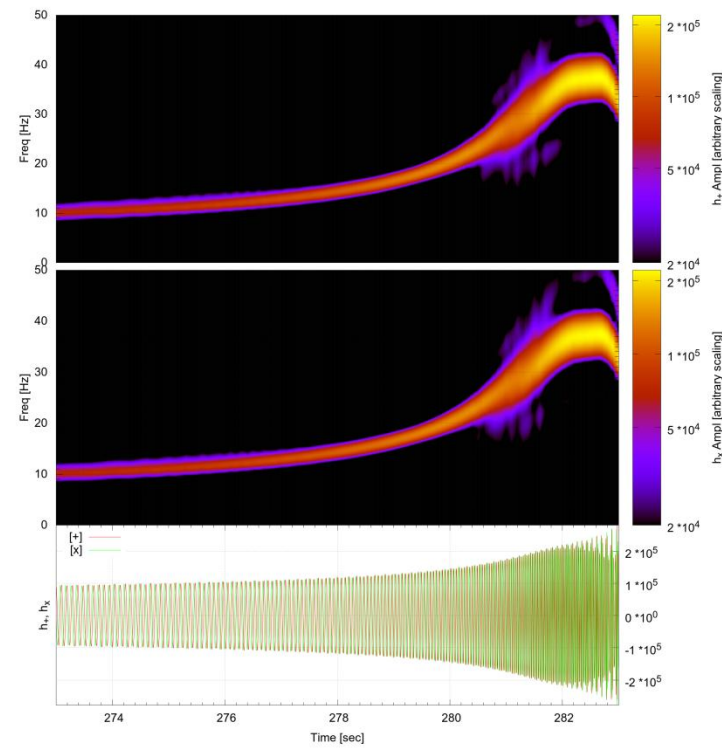
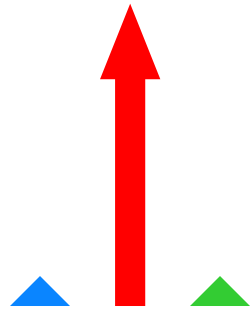
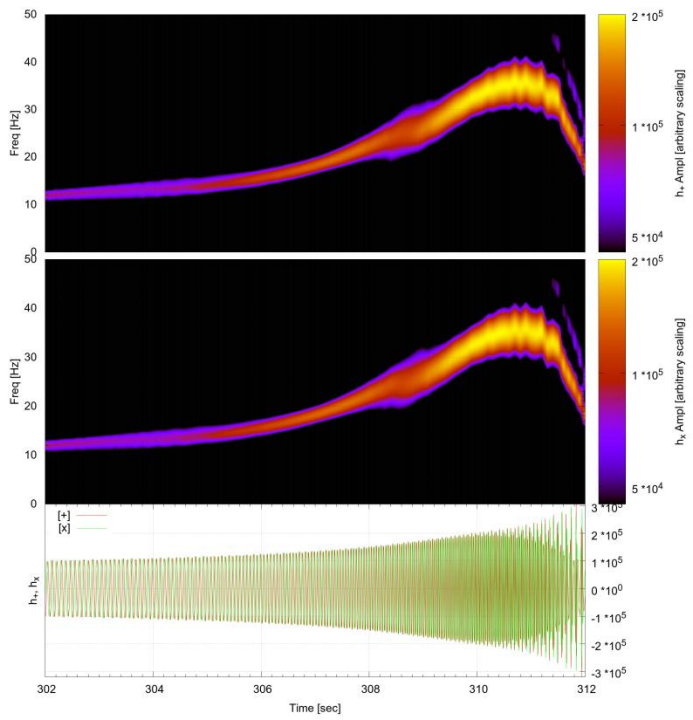
101160 - 100975



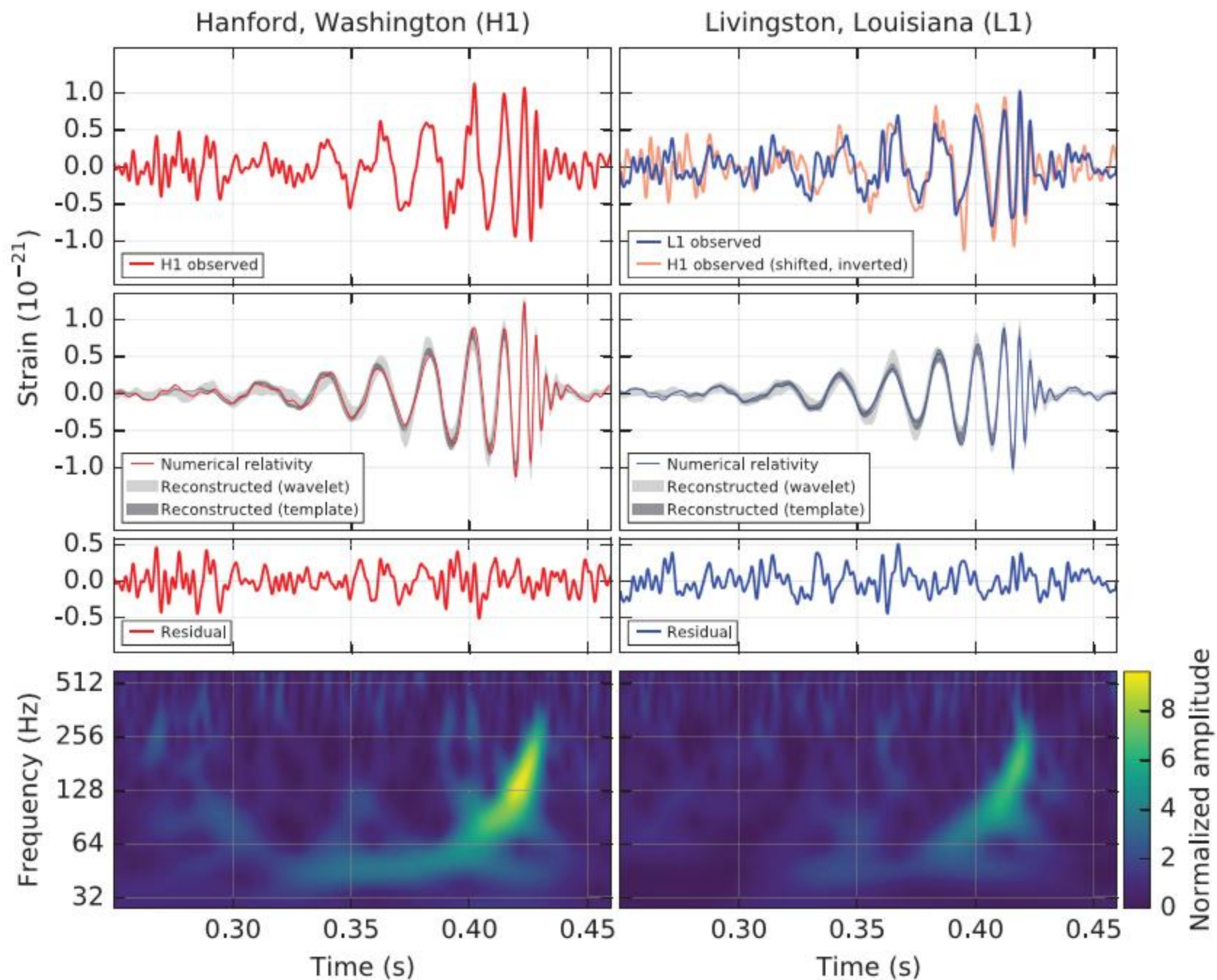
Berczik et al. (in prep.)







Berczik et al. (in prep.)



Conclusions:

- With our N-body⁶⁺⁺GPU code we are already (sub)exascale “ready”!**
- The GPU accelerator is critical for the high performance. We should use more data “calculation” instead of “moving” data (between nodes!).**
- The current GPU supercomputers already allow us to perform the large scale ($\sim 1\text{M}$) simulation of the realistic GC with needed accuracy for the “real” BBH merging.**

Thank you for your attention...

